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State-of-the-art review of inherent variability and uncertainty in geotechnical properties and models

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STATE-OF-THE-ART REVIEW

OF

INHERENT VARIABILITY AND UNCERTAINTY

IN

GEOTECHNICAL PROPERTIES AND MODELS



March 2, 2021

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Technical Committee of Engineering Practice of Risk Assessment & Management (TC304)

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Preface

CEN committee TC250 is currently working on an update of the Eurocodes. Sub-committee SC10, in charge of updating EN 1990 (Basis of structural and geotechnical design), has installed a working group to produce a background document with the working title 'Reliability Backgrounds of the Eurocodes', with the intention to document and explain the reliability framework underlying all Eurocodes and the implementation of reliability aspects in them. As part of that effort, quantitative information on the inherent variability and uncertainty in loads, material properties and models is compiled. ISSMGE-TC304 identified this as an opportunity to provide an overview of the relevant information available in the geotechnical literature such as the statistics of soil/rock properties. The EPRI TR-105000 report (Phoon et al. 1995) provided an overview of the statistics of inherent soil properties and measurement errors, but these statistics have not yet been updated systematically since 1995. Also, rock properties were not covered by the TR-105000 report. Other than soil/rock properties and measurement errors, there are also other important statistics, such as the statistics of transformation uncertainties and model factors.

The current technical report is entitled "State-of-the-Art Review of Inherent Variability and Uncertainty in Geotechnical Properties and Models". It contains the following seven chapters as shown in the following table.

		1 1
Chap	Title	Contributors
1	Site-specific statistics for	Zheng Guan, Yu-Chi Chang, Yu Wang (lead), Adeyemi Aladejare,
	geotechnical properties	Dongming Zhang, and Jianye Ching
2	Site-specific correlations between	Yelu Zhou, Dongming Zhang (lead), and Jianye Ching
	soil/rock properties	
3	Summary of random field	Armin W. Stuedlein (lead), Brigid Cami, Diego Di Curzio, Sina
	parameters of geotechnical	Javankhoshdel, Shin-ichi Nishimura, Wojciech Pula, Giovanna
	properties	Vessia, Yu Wang, and Jianye Ching
4	Statistics for geotechnical design	Chong Tang (lead) and Richard Bathurst
	model factors	
5	Statistics for transformation	Jianye Ching (lead) and Ali Noorzad
	uncertainties	
6	Determining characteristic values	Zi-Jun Cao (lead), Jianye Ching, Guo-Hui Gao, Mikhail
	of geotechnical parameters and	Kholmyansky, Ali Noorzad, Timo Schweckendiek, Johan Spross,
	resistance: an overview	Mohammad Tabarroki, Xiaohui Tan, Yu Wang, Tengyuan Zhao,
		and Yan-Guo Zhou
7	Numerical evidences for	Giovanna Vessia (lead), Yan-Guo Zhou, Andy Leung, Wojciech
	worst-case scale of fluctuation	Pula, Diego Di Curzio, Mohammad Tabarroki, and Jianye Ching

Table P1. Titles of the seven chapters in the report

The current technical report has the following features:

- 1. It serves as an update for the TR-105000 report on the statistics of inherent soil properties. Chapter 1 compiles the site-specific statistics for univariate soil properties. Chapter 3 compiles the random field parameters (e.g., the scales of fluctuation) for spatial variability of soils. Many of the statistics are new.
- 2. It contains statistics that are not covered by the TR-105000 report. Chapter 1 compiles the

State-of-the-art review of inherent variability and uncertainty, March 2021 site-specific statistics for some rock and rock mass properties. Chapter 2 compiles the site-specific correlations between soil/rock properties. Chapter 5 compiles the statistics for transformation uncertainties.

3. Chapter 4 compiles the statistics of geotechnical design model factors. Chapter 6 reviews methods that determine the characteristic value defined by the Eurocode 7. Chapter 7 reviews some numerical evidences for the worst-case scale of fluctuation.

Many of the new updates in #1 and #2 above are based on the databases in 304dB, an open-access database sharing initiative developed by ISSMGE TC304:

http://140.112.12.21/issmge/tc304.htm?=6

While updating of the Eurocodes triggered the work on the present state-of-the-art review, we trust that the information contained will be a valuable resource for other codes of practice as well as for researchers and practitioners in the field of geotechnical reliability.

We would like to acknowledge the tremendous efforts contributed by the seven groups of experts. This report would not be possible without their efforts.

Editors Jianye Ching (Chair of TC304, ISSMGE) Timo Schweckendiek (member of TC304, ISSMGE)

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Phoon, K.K., Kulhawy, F.H., and Grigoriu, M.D. (1995). Reliability-Based Design of Foundations for Transmission Line Structure, Report TR-105000, Palo Alto, Electric Power Research Institute.

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1. Site-specific statistics for geotechnical properties

Zheng Guan, Yu-Chi Chang, Yu Wang, Adeyemi Aladejare, Dongming Zhang, and Jianye Ching

1.1 Introduction

The proper characterization of the variability of geotechnical properties for a specific site plays a critical role in reliability-based design (RBD) of geotechnical structures (e.g., Phoon and Kulhawy 1999; Baecher and Christian 2003; Fenton and Griffiths 2008; Cao et al. 2016). However, in geotechnical site investigation, site-specific measurement data are usually sparse and limited, particularly for small or medium-sized projects (e.g., Wang and Cao 2013). This leads to the difficulty in obtaining meaningful site-specific statistics (e.g., mean, μ , and coefficient of variation, COV) of geotechnical properties from site-specific measurement data. To deal with these challenges, sparse site-specific data might be integrated with prior knowledge such as typical ranges of μ and COV (e.g., Phoon and Kulhawy 1999; Wang and Cao 2013). This underlines a need to summarize the typical values of site-specific μ and COV from previous studies and reports.

The EPRI TR-105000 report (Phoon et al. 1995), denoted by TR-105000 later, complied statistics of some soil properties from the literature. Since then, soil property statistics have not been systematically updated. Nonetheless, some soil/rock databases have been collected recently, as shown in Table 1.1. The purpose of the current report is to extract site-specific statistics from these databases to serve as an update for TR-105000.

Database	Reference	Parameters of interest	# data points	# sites/ studies
CLAY/10/7490	Ching and Phoon (2014)	LL, PI, LI, σ'_{ν}/P_a , σ'_p/P_a , s_u/σ'_{ν} , S_t , q_{t1} , q_{tu} , B_q	7490	251 studies
SAND/7/2794	Ching et al. (2017)	$D_{50}, C_u, D_r, \sigma'_v / P_a, \phi', q_{c1n}, (N_1)_{60}$	2794	176 studies
ROCK/13	Aladejare and Wang (2017)	$\rho,G_s,I_{d2},n,w,\gamma,R_L,S_h,\sigma_{bt},I_{s50},\sigma_{ci},E_i,\nu$		
ROCK/9/4069	Ching et al. (2018)	$\gamma,n,R_L,S_h,\sigma_{bt},I_{s50},V_p,\sigma_{ci},E_i$	4069	184 studies
ROCKMass/9/5876	Ching et al. (2020)	RQD, RMR, Q, GSI, E_m , E_{em} , E_{dm} , E_i , σ_{ci}	5784	225 studies
CLAY/8/12225	Ching (2020)	LL, PI, w, e, $\sigma' v/P_a$, C _c , C _{ur} , c _v	12225	427 studies
CLAY/12/3997	Ching (2020)	LL, PI, LI, σ'_{ν} /P _a , σ'_{p} /P _a , $s_{u'}\sigma'_{\nu}$, K ₀ , $E_{u'}\sigma'_{\nu}$, B _q , q _{t1} , N ₆₀ /(σ'_{ν} /P _a)	3997	237 studies
SAND/13/4113	Ching (2020)	$\begin{array}{c} e,D_r,\sigma'_\nu\!/P_a,\sigma'_p\!/P_a,K_0,E_{dn},q_{c1n},B_q,(N_1)_{60},K_{DMTn},\\ E_{DMTn},E_{PMTn},M_{dn} \end{array}$	4113	172 studies
SH-CLAY/11/4051	Zhang et al. (2020)	LL, PI, LI, e, K ₀ , σ'_v/P_a , $s_{u(UCST)}/\sigma'_v$, $s_{u(VST)}/\sigma'_v$, Su(UCST), SU(VST), p_s/σ'_v	4051	50 sites in Shanghai

 Table 1.1. Soil/rock databases

 ρ = density; v = Poisson ratio; γ = unit weight; ϕ' = effective friction angle; σ_{p}° = preconsolidation stress; σ_{v}° = vertical effective stress; σ_{bt} = Brazilian tensile strength; σ_{ci} = uniaxial compressive strength of intact rock; $(N_{1})_{60} = N_{60}/(\sigma'_{v}/P_{a})^{0.5}$; B_{q} = CPT pore pressure ratio = $(u_{2}-u_{0})/(q_{t}-\sigma_{v})$; C_{c} = compression index; C_{ur} = unload/reload index; C_{u} = coefficient of uniformity; c_{v} = coefficient of consolidation; Dso = median grain size; Dr = relative density; end units determined by PMT; E_{d} = drained modulus of sand; EpMTn = normalized EpMTr = (EpMT/Pa)/(σ'_{v}/Pa)^{0.5}; Edm = (Ed/Pa)/(σ'_{v}/Pa)^{0.5}; Edm = dynamic modulus of rock mass; Eue = elasticity modulus of rock mass; Ei = Young's modulus of intact rock; Em = deformation modulus of rock mass; Eue = undrained modulus of clay; G_{s} = specific gravity; GSI = geological strength index; Id_{2} = slake durability index; Is_{50} = point load strength index; LL = liquid limit; n = porosity; M_{d} = effective constrained modulus determined by oedome

State-of-the-art review of inherent variability and uncertainty, March 2021 To extract reliable statistics, only sites in the databases with more than 10 data points are used. There are also sites in the databases with more than 30 data points. The site-specific statistics for these sites are considered to be very reliable. In Table 1.1, there is a municipal database of Shanghai (SH-CLAY/11/4051).

1.2 Data Tables

The data tables for site-specific statistics of clay, sand, and rock properties are shown in the Appendix. These site-specific statistics are extracted from the databases in Table 1.1. The site-specific statistics in TR-105000 are not included in these data tables.

1.3 Summary Figures

Based on the data tables in the Appendix, summary figures for clay, sand, and rock are developed. These figures show the distributions of site-specific statistics. Site-specific statistics for more than 30 data points are marked as red, whereas those for 10-30 data points are marked as yellow. The city-specific statistics for this municipal database are shown as blue crosses 'x'. For comparison, the results for TR-105000 are also shown in the figures as grey triangles.



1.3.1 Figures for clay properties

Figure 1.1. Statistics for Atterberg limits (PL and LL) of clays



Figure 1.2. Statistics for water content (w) and liquidity index (LI) of clays



Figure 1.3. Site-specific statistics for plasticity index (PI) of clays



Figure 1.4. Statistics for sensitivity (St) of clays



Figure 1.5. Statistics for overconsolidation ratio (OCR) of clays



Figure 1.6. Statistics for compression (C_c) and unload-reload (C_{ur}) indices of clays



Figure 1.7. Statistics for undrained strength (s_u) and normalized strength (s_u/σ'_v) of clays

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Figure 1.8. Statistics for CPT tip resistance $(q_c \text{ or } q_t)$ and normalized tip resistance (q_{t1}) of clays



Figure 1.9. Statistics for CPT pore pressure coefficient (B_q) of clays



Figure 1.10. Statistics for SPT blow count (N) of clays



Figure 1.11. Statistics for EDMT, EPMT, and Md of clays



Figure 1.12. Statistics for K₀ and K_{DMT} of clays

1.3.2 Figures for sand properties



Figure 1.13. Statistics for void ratio (e) of sands



Figure 1.14. Site-specific statistics for friction angle (ϕ) of sands and clays



Figure 1.15. Statistics for SPT blow count (N) of sands



Figure 1.16. Statistics for tip resistance (qc) and normalized tip resistance (qc1n) of sands



Figure 1.17. Statistics for EDMT and EPMT of sands



Figure 1.18. Statistics for K₀ and K_{DMT} of sands

1.3.3 Figures for intact rock properties



Figure 1.19. Statistics for unit weight (γ) and porosity (n) of intact rocks



Figure 1.20. Statistics for hardnesses (R_L and S_h) and strengths (σ_{bt} and I_{s50}) of intact rocks



Figure 1.21. Statistics for uniaxial compressive strength (σ_{ci}) and Young's modulus (E_i) of intact rocks



Figure 1.22. Statistics for P-wave velocity (V_p) of intact rocks

1.3.4 Figures for rock mass properties



Figure 1.23. Statistics for RQD of rock masses



Figure 1.24. Statistics for RMR, GSI, and Q of rock masses



Figure 1.25. Statistics for deformation modulus (E_m) of rock masses

1.4 Summary Tables

Based on the site-specific statistics in the Appendix and those in TR-105000, summary tables (Tables 1.2, 1.3, and 1.4) for clay, sand, and rock are developed. These summary tables summarize the mean and range of the site-specific statistics. For example, the first item in Table 1.2 indicates that the site-specific COV for LL of clay ranges from 3.4-39% with mean = 15.6%. Note that these tables are developed based on the combined results of Appendix and TR-105000.

Durant	#	# cases/	group	Site-	Site-specific mean			Site-specific COV		
Property	groups	Range	Mean	Range	Mean	95% CI	Range	Mean	95% CI	
LL (%)	103	10-2229	69	19.3-158.6	55.6	24.7-95.1	3.4-39	15.6	4.8-35.1	
PL(%)	87	10-299	41	13.9-112.7	29.1	17.2-76.2	2.9-38.1	13.5	3.9-35.0	
PI (%)	94	10-4044	93	6.2-60.8	29.0	10.5-56.2	6.5-57	23.5	6.8-47	
w (%)	111	10-439	76	13.1-120.2	43.5	13.7-104.9	3.5-46	15.3	4.9-30	
LI	49	10-2067	68	0.09-2.47	0.93	0.09-2.31	5.8-88	24.5	5.8-70.5	
OCR	24	10-56	17	0.90-3.15	1.69	0.90-3.11	1.2-39	17.8	1.5-38.8	
Cc	18	17-136	53	0.19-2.15	0.63	0.19-2.15	18.1-47.3	35.6	18.1-47.3	
C_{ur}	9	17-115	44	0.03-0.21	0.10	0.03-0.21	22.6-50.5	42.4	22.6-50.5	
φ (°)	13	5-51	19	3-33.3	15.3	3-33.3	10-50	21.3	10-50	
s _u (kPa)	91	9-393	59	6.3-712.8	148.0	7.2-558.4	6-56	28.2	9.9-53.5	
s_u/σ'_v	45	10-352	27	0.05-1.14	0.39	0.06-1.07	3.2-39.4	20.8	5.0-39.3	
\mathbf{S}_{t}	17	10-384	51	2.2-38.6	8.8	2.2-38.6	12.4-63.4	30.8	12.4-63.4	
q_{c}	11	47-53	50	1.2-2.1	1.65	1.2-2.1	16-40	28.4	16-40	
q_{t}	9	-	-	0.4-2.7	1.54	0.4-2.7	2-17	7.9	2-17	
q_{t1}	21	12-42	17	2.04-13.2	5.99	2.04-13.13	5.8-39.8	17.5	5.8-39.7	
$\mathbf{B}_{\mathbf{q}}$	26	11-47	20	0.17-0.99	0.57	0.18-0.96	6.5-58.3	20.3	6.6-55.8	
SPT-N	11	12-61	27	1.75-75.3	33.0	1.75-75.3	15.9-57	30.7	15.9-57	
E _{DMT} (MPa)	25	10-32	17	0.71-33.7	7.2	0.76-32.36	4.6-45.8	24.0	5.3-56.6	
E _{PMT} (MPa)	4	10-22	15	22.1-160.6	68.0	22.1-160.6	19.8-39.1	29.3	19.8-39.1	
M _d (MPa)	5	10-13	11	0.49-4.60	2.66	0.49-4.60	20.8-46.8	34.6	20.8-46.8	
K ₀	8	10-264	45	0.48-2.88	1.28	0.48-2.88	2.4-22	13.5	2.4-22.0	
K _{DMT}	47	10-50	18	1.34-15.12	3.91	1.70-12.69	6.2-49.4	18.2	6.3-40.6	

Table 1.2. Summary of site-specific statistics for clay

Table 1.3. Summary of site-specific statistics for sand

Dreamenter	#	# cases/group		Site-specific mean			Site-specific COV		
Property	groups	Range	Mean	Range	Mean	95% CI	Range	Mean	95% CI
e	6	11-17	14	0.47-0.63	0.55	0.47-0.63	7-19.9	11.1	7-19.9
φ (°)	23	10-136	32	32.3-52	38.4	32.4-51.5	4.2-12.5	7.9	4.3-12.4
q_{c}	49	10-2039	125	0.7-26	3.3	0.85-13.17	17-81	39.7	17.0-77.4
q_{c1n}	25	10-28	15	14.1-254.6	90.4	14.2-247.4	11.5-68	36.9	11.9-68
SPT-N	26	10-300	62	6.8-74	32.9	6.8-73.3	18.4-62	34.3	18.5-61.0
(N ₁) ₆₀	9	11-35	21	5.7-28.6	15.3	5.7-28.6	16.5-38.8	32.2	16.5-38.8
E _{DMT} (MPa)	53	10-25	14	2.21-71.4	26.2	5.63-62.0	7-92	37.0	8.7-73.0
E _{PMT} (MPa)	7	10-53	26	5.24-26.1	12.6	5.24-26.1	15.7-68	34.3	15.7-68
K_0	4	13-15	15	0.64-2.20	1.16	0.64-2.20	25.8-36.9	33.1	25.8-36.9
K _{DMT}	15	10-25	15	1.9-28.3	15.1	1.9-28.3	20-99	44.3	20-99

	#	# cases/	group	Site-	specific	mean	Site-specific COV		
Property	groups	Range	Mean	Range	Mean	95% CI	Range	Mean	95% CI
n (%)	31	10-262	38	0.2-36.2	6.9	0.2-33.1	1.5-115.1	50.1	2.7-114.7
γ (kN/m ³)	56	10-778	44	5.4-30.1	24.6	18.0-28.1	0.4-21.5	5.2	0.6-18.5
V _P (km/s)	32	10-27	15	0.81-6.03	3.90	1.20-5.97	1.47-44.7	14.1	2.1-40.7
R _L	23	10-355	53	26.3-62.6	39.9	26.3-62.2	3.0-37.4	19.1	3.2-37.1
S_h	9	11-31	22	13.4-76.1	47.0	13.4-76.1	8.1-35.3	19.1	8.1-35.3
Is50 (MPa)	58	10-1305	63	0.17-9.04	3.69	1.21-9.02	5.1-91.5	34.4	5.1-91.4
σ _{bt} (MPa)	31	10-43	18	2.35-19.4	9.23	3.2-19.4	6.6-64.5	25.8	6.6-61.7
σ _{ci} (MPa)	116	10-470	29	1.9-226.9	66.6	8.7-151.2	5.7-108.4	33.8	6.6-84.1
E _i (GPa)	53	10-99	26	0.13-85.9	24.37	0.53-77.49	3.8-73.7	33.4	3.8-67.6
RQD	43	10-80	21	25.6-95.8	65.6	26.3-92.8	4.8-114.8	29.9	5.5-108.9
RMR	55	10-330	31	20.3-81.2	53.7	25.2-81.2	4.7-46.8	21.3	6.2-39.1
GSI	22	10-111	23	13.6-64.5	44.4	14.0-64.2	3.0-57.0	19.9	3.1-56.4
Q	26	10-28	18	0.13-74.28	11.7	0.16-70.17	17.6-303.5	104.7	19.4-289.4
E _m (GPa)	16	10-28	19	0.11-35.1	13.6	0.11-35.1	14.7-103.0	55.6	14.7-103.0

State-of-the-art review of inherent variability and uncertainty, March 2021 **Table 1.4.** Summary of site-specific statistics for rock and rock mass

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1.6 Appendix: data tables for site-specific statistics of clay, sand, and rock properties

Source	Site Description		Property	No. of tests (≥ 30)	Range of data	Property mean	Property COV (%)
		c (kPa)	Cohesion	42	4.0-29.0	13.7	38.0
Wang and Zhu 2016	2010 Expo Park in Shanghai (Silty clay)	φ (°)	Friction angle	42	8.4-32.5	21.1	22.7
		E (MPa)	Young's modulus	42	2.3-13.9	5.2	51.9
		w (%)	Natural moisture content	56	26.6-48.1	36	14.3
Chin et al.	Taipei City Hall Station	LL (%)	Liquid limit	56	21.9-46.8	36	16.4
1994	(Clay)	PI (%)	Plasticity index	56	5.9-29.1	14.9	35.0
		$\gamma_t (kN/m^3)$	Total unit weight	56	17.9-20.9	19.0	3.3
		LL(%)	Liquid limit	2229	26.3-58.7	40.3	22
		PL (%)	Plastic limit	2350	13.8-43.50	22.25	9
	-	PI (%)	Plasticity index	4044	10.3-30.9	18.2	18
	_	w (%)	Water content	4011	23.30-63.60	43.29	15
	Shanghai Clay Database SH-CLAY/11/4051	LI	Liquidity index	2067	0.49-2.19	1.15	22
		e	Void ratio	3875	0.67-1.86	1.24	14
Zhang et al. (2020)		K_0	At-rest lateral pressure coefficient	264	0.43-0.65	0.51	9
	-	s_u/σ'_v	Normalized UC	148	0.10-0.59	0.19	34
	-	St	UC	181	1.5-7.6	5.2	28
	-	s_u/σ'_v	Normalized VST	352	0.22-0.71	0.31	23
	-	St	VST	384	2.7-7.8	3.95	17
	-	$p_s\!/\sigma'_{\nu}$	Specific penetration resistance ratio	1148	2.55-46.05	5.71	45
Gregersen and	Baastad (Norway)	w (%)	Natural moisture content	32	25.0-35.7	30.1	9.8

Table 1.A1. Site-specific clay property statistics (databases: CLAY/10/7490, CLAY/8/12225, CLAY/12/3997 & SH-CLAY/11/4051)
			Ju	ale-or-life-art	Teview of innerent v	anability and unco	ertainty, March 202
Løken (1979)		LL(%)	Liquid limit	30	19.1-43.2	31.9	24.6
		PL(%)	Plastic limit	30	11.2-28.7	21.2	21.8
		PI (%)	Plasticity index	30	5.0-17.5	10.7	36.6
		St	Sensitivity	21	1.5-6.0	3.7	35.7
	_	w (%)	Natural moisture content	47	27.7-74.0	60.1	15.9
		LL(%)	Liquid limit	47	60.5-96.6	83.5	10.4
Hanzawa		PL(%)	Plastic limit	47	25.2-60.8	38.6	14.2
(1979)	Natsushima (Japan) —	PI (%)	Plasticity index	47	35.3-54.7	44.9	11.5
		LI	Liquid index	47	0.8-1.0	0.9	5.8
	-	s_u / σ'_v	Normalized VST	17	0.50-0.67	0.60	7.7
		w (%)	Natural moisture content	31	28.9-57.8	48.4	15.5
		LL(%)	Liquid limit	31	24.5-51.2	38.3	16.3
		PL(%)	Plastic limit	31	16.7-24.8	21.2	9.3
Ladd (1972)	Portsmouth, New — Hampshire (USA) — —	PI (%)	Plasticity index	31	7.8-26.8	17.1	25.8
		LI	Liquid index	28	1.33-2.53	1.76	15.2
		s_u/σ'_v	Normalized VST	19	0.18-0.31	0.24	19.2
		\mathbf{S}_{t}	Sensitivity	18	7.3-12.9	9.5	18.6
		w (%)	Natural moisture content	55	32.4-50.5	40.8	13.8
		LL(%)	Liquid limit	54	40.8-60.0	51.1	10.5
		PL(%)	Plastic limit	54	20.8-29.2	23.9	9.7
Bjerrum and	<u> </u>	PI (%)	Plasticity index	54	19.2-31.7	27.1	12.9
Lo (1963)	Skabo, Oslo (Norway) —	LI	Liquid index	54	0.3-0.8	0.6	21.9
	—	s _u (kPa)	VST	42	21.4-37.8	28.8	14.9
	—	s _u (kPa)	UC	20	21.7-45.9	30.4	19.4
	—	\mathbf{S}_{t}	Sensitivity	48	2.6-8.9	5.2	23.5
陳厚銘 and	Taipei	w (%)	Natural moisture content	23	25.5-40.8	36.2	10.6
謝百鍾	(Taiwan)	LL(%)	Liquid limit	23	30.1-48.4	39.4	12.3

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(1996)		PL(%)	Plastic limit	23	19.5-24.6	22.6	6.6
		PI (%)	Plasticity index	23	8.8-23.7	16.8	22.5
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	23	0.10-0.77	0.61	27.7
		OCR	Over consolidation ratio	17	1.13-1.53	1.30	10.2
		w (%)	Natural moisture content	60	50-95	67.5	18.4
		LL(%)	Liquid limit	60	68-131	91.7	18.6
Jacob and G	Wettened (India)	PL(%)	Plastic limit	60	29-69	46.2	16.1
(2016)	Kuttanad (India) —	PI (%)	Plasticity index	60	27-62	45.6	23.1
		LI	Liquid index	60	0.3-0.7	0.5	17.6
		Cc	Compression index	60	0.38-0.79	0.57	18.2
		w (%)	Natural moisture content	40	20.5-36.8	28.4	15.7
		LL(%)	Liquid limit	39	30.0-45.5	38.7	15.5
Kinner (1970)	Boston	PL(%)	Plastic limit	39	17.5-23.9	20.1	12.0
	(0011)	PI (%)	Plasticity index	39	12.5-22.5	18.6	22.2
		LI	Liquid index	36	0.24-0.61	0.42	20.1
		w (%)	Natural moisture content	35	14.9-27.7	22.5	16.5
		LL (%)	Liquid limit	30	32.7-56.4	41.5	12.9
		PL(%)	Plastic limit	30	16.0-23.9	19.4	8.3
Lunne et al.	Sleipner Area	PI (%)	Plasticity index	30	15.9-32.6	22.2	18.2
(1985)	(Norway)	LI	Liquid index	30	-0.11-0.35	0.19	60.6
		$\mathbf{B}_{\mathbf{q}}$	CPTu pore pressure parameter	32	-0.03-0.63	0.34	58.3
		q_{t1}	Normalized cone tip resistance	26	2.64-9.92	4.68	39.8
Mayne (1991)	Jamestown, Virginia (USA)	w (%)	Natural moisture content	33	27.7-43.4	35.2	10.7
D.		LL(%)	Liquid limit	32	23.1-41.4	31.4	19.3
Bjerrum (1954)	Toyon, Oslo (Norway)	PL(%)	Plastic limit	32	16.4-26.6	20.9	11.1
(LI	Liquid index	32	0.6-3.1	0.8	33.0

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		w (%)	Natural moisture content	37	26.5-41.3	35.5	10.6
	-	LL(%)	Liquid limit	37	19.4-39.0	24.8	18.0
	Manglerud (Norway)	PL(%)	Plastic limit	37	14.9-24.8	18.1	12.6
	-	s_u/σ'_v	Normalized VST	20	0.05-0.22	0.13	33.4
	-	s_u/σ'_v	Normalized UC	13	0.03-0.17	0.05	17.9
		w (%)	Natural moisture content	38	28.5-44.7	35.2	13.7
		LL(%)	Liquid limit	38	31.8-45.8	38.1	8.6
	-	PL(%)	Plastic limit	38	18.2-24.8	21.2	8.3
	Drammen (Norway)	PI (%)	Plasticity index	38	9.4-23.4	16.9	20.5
	-	LI	Liquid index	35	0.5-1.38	0.80	29.1
		$\mathbf{B}_{\mathbf{q}}$	CPTu pore pressure parameter	40	0.45-0.90	0.71	16.1
	 Mid-Atlantic (USA) 	LL(%)	Liquid limit	48	22.0-61.0	43.3	23.2
		PL(%)	Plastic limit	48	15.0-30.0	22.5	20.5
Oleon(1082)		PI (%)	Plasticity index	48	6.0-34.0	20.4	29.3
Olsell (1982)		LI	Liquid index	48	0-3.7	1.3	43.0
		Cc	Compression index	40	0.1-0.7	0.38	39.8
		C_{ur}	Swell index	40	0.01-0.11	0.06	44.3
		w (%)	Natural moisture content	33	54.7-80.4	64.6	9.7
		LL(%)	Liquid limit	33	64.1-104.2	87.8	9.8
		PL(%)	Plastic limit	33	24.5-41.6	33.0	16.9
Cao et al.	Singanora	PI (%)	Plasticity index	33	39.6-70.2	54.8	13.0
(2001)	Singapore –	LI	Liquid index	33	0.24-0.98	0.59	29.2
	-	OCR	Over consolidation ratio	30	1.12-2.37	1.67	34.6
	-	Cc	Compression index	46	0.56-1.27	0.87	18.1
	-	Cur	Swell index	46	0.05-0.25	0.16	22.6
Chung et al.	Busan	w (%)	Natural moisture content	55	23.2-77.3	53.5	26.0

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(2005)	(South Korea)	LL(%)	Liquid limit	55	30.9-74.3	54.6	20.6
		PL(%)	Plastic limit	55	14.6-55.7	28.1	26.8
		PI (%)	Plasticity index	55	10.7-40.0	26.6	27.7
		LI	Liquid index	55	0.19-2.04	0.97	38.4
		s_u/σ'_v	Normalized VST	26	0.28-0.70	0.39	25.6
		OCR	Over consolidation ratio	56	0.13-1.60	0.90	36.6
		s_u/σ'_v	Normalized UC	46	0.04-0.56	0.29	39.0
	_	Cc	Compression index	56	0.15-1.05	0.55	40.7
		w (%)	Natural moisture content	33	32.0-78.0	47.3	26.6
		LL(%)	Liquid limit	33	34.0-78.0	50.7	21.4
Munshi		PL(%)	Plastic limit	33	13.0-50.0	26.7	23.2
(2003)	Bangladesh –	PI (%)	Plasticity index	33	7.0-51.0	23.9	35.6
		LI	Liquid index	29	-0.332-1.89	0.75	63.9
		Cc	Compression index	33	0.19-0.65	0.32	30.4
	— Santa Barbara — Channel,	w (%)	Natural moisture content	32	32.3-86.5	47.3	24.0
		LL(%)	Liquid limit	31	48.5-84.1	62.3	11.5
Quiros and Young		PL(%)	Plastic limit	32	28.5-37.4	32.2	6.6
(1988)	California	PI (%)	Plasticity index	31	16.0-48.1	30.3	19.2
	(USA) —	s_u/σ'_v	Normalized VST	15	0.3-0.7	0.49	28.0
	_	s_u/σ'_v	Normalized UU	10	0.32-0.69	0.44	24.0
		LL(%)	Liquid limit	35	50.2-74.4	63.5	11.6
	_	PL(%)	Plastic limit	35	29.0-38.3	34.6	5.9
		LI	Liquid index	35	0.7-1.8	1.0	21.8
Lacasse and Lunne (1982)	Norway	s_u/σ'_v	Normalized VST	30	0.2-0.5	0.3	21.4
Lunie (1902)	_	w (%)	Natural moisture content	45	57.3-70.0	63.4	6.0
		LI	Liquid index	40	0.42-1.14	0.74	22.7
		$\mathbf{B}_{\mathbf{q}}$	CPTu pore pressure	47	0.3-0.7	0.5	19.1

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			parameter				
	-	q_{t1}	Normalized cone tip resistance	42	4.92-6.67	5.83	5.8
Cooling and Skempton (1942)	London (UK)	PL (%)	Plastic limit	100	21.7-31.9	26.7	9.4
Carrassco et al. (2004)	Madrid (Spain)	SPT-N	Standard penetration test blow count	40	42.3-95.9	75.3	15.9
Totani et al. (1998)	large clay waste disposal at Santa Barbara open-pit mine (center Italy)	E _{DMT} (MPa)	Modulus from DMT	32	8.69-18.15	12.22	20.2
I_{in} (1000)	Toinoi (Toiwan)	PI (%)	Plasticity index	48	5.76-25.46	15.23	31.9
Liu (1999)	Taiper (Taiwaii)	s_u/σ'_v	Normalized CIUC	43	0.15-0.37	0.24	26.1
Chin et al.	Tainai (Taiwan)	w (%)	Natural water content	32	23.7-52.6	38.2	22.9
(1989)		PI (%)	Plasticity index	32	7.6-26.2	16.4	36.2
	- - Amberst	w (%)	Natural water content	49	56.6-74.0	64.0	6.1
		LL(%)	Liquid limit	49	39.6-55.5	51.1	7.6
		PL(%)	Plastic limit	46	27.8-32.3	30.9	4.1
DeGroot and		PI (%)	Plasticity index	49	11.8-25.0	20.7	14.5
Lutenegger (2002)	Massachusetts (USA)	LI	Liquidity index	49	1.2-2.9	1.7	21.6
(2002)	-	Bq	CPTu pore pressure parameter	32	0.63-0.96	0.75	13.1
		\mathbf{S}_{t}	Sensitivity	47	4.8-23.2	10.6	47.2
		s_u/σ'_v	Normalized VST	50	0.23-0.68	0.38	26.6
		w (%)	Natural water content	42	70.8-90.0	78.4	5.9
		LL(%)	Liquid limit	46	60.5-96.6	83.2	8.1
Hanzawa	Notouchimo (Ionon)	PL(%)	Plastic limit	42	32.4-42.5	37.7	6.9
(1979)	Thatsushinna (Japan) –	PI (%)	Plasticity index	41	37.8-54.7	45.4	10.4
	-	LI	Liquidity index	46	0.76-0.99	0.89	5.8
	-	s_u/σ'_v	Normalized VST	18	0.50-0.67	0.60	7.5

		w (%)	Natural water content	33	25.0-29.4	27.1	5.0
Agarwal (1967)	Wraysbury (UK)		Normalized CIUC	21	0.16-0.29	0.23	15.8
(1907)		S_u/O_v	Normalized UU	12	0.39-1.24	0.71	32.8
Cozzolino	Cozzolino (1961) Santos (Spain)		Liquid limit	35	47.1-136.4	95.4	26.3
(1961)	Santos (Spani) —	Cc	Compression index	52	0.34-1.75	0.95	37.4
		LL(%)	Liquid limit	26	27-70	48.6	22.3
Bartlett and	Salt Lake City, Utah	PL(%)	Plastic limit	26	9-45	22.1	37.2
Lee (2004)	(USA)	OCR	Over consolidation ratio	27	0.66-2.68	1.33	39.0
		Cc	Compression index	35	0.13-1.03	0.50	42.2
Finno and	Chienge (USA)	w (%)	Natural water content	47	18.4	52.6	28.1
Chung (1992)	Chicago (USA)	OCR	Over consolidated ratio	16	0.81-2.41	1.57	34.9
Briaud et al.	Houston (USA) —	Cc	Compression index	32	0.06-0.48	0.26	43.3
(2007)		C _{ur}	Swell index	30	0.03-0.15	0.08	48.4
Liu et al.	Lianyungang (China) —	w (%)	Natural water content	136	30.3-753	52.3	19.2
(2011)		C_c	Compression index	136	0.20-1.03	0.59	29.0
Giao and Hien	Red River Delta (Vietnam)	Cc	Compression index	115	0.07-0.33	0.19	36.5
(2007)		Cur	Swell index	115	0.006-0.106	0.04	45.8
		LL(%)	Liquid limit	15	66.9-257.3	158.6	37.9
		PL(%)	Plastic limit	12	48.2-182.6	112.7	38.1
Baroni et al.	Jacarepaguá Lowlands	PI (%)	Plasticity index	12	18.8-96.3	49.6	44.4
(2017)	(Brazil)	\mathbf{S}_{t}	Sensitivity	21	1.22-13.41	7.26	50.5
		Cc	Compression index	44	0.35-4.22	2.15	40.9
		Cur	Swell index	28	0.06-0.47	0.21	48.1
Tan et al.	Vlang (Malaycia)	w (%)	Natural water content	65	90.5-136.3	97.0	29.3
(2003)	Klang (Malaysia)	Cc	Compression index	63	0.23-2.39	1.25	42.9
Long et al.	Belfast (Northern	Cc	Compression index	40	0.08-1.26	0.63	46.3
(2007)	Ireland)	C _{ur}	Swell index	38	0.01-0.26	0.12	49.5

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		w (%)	Natural water content	60	50-95	67.5	18.4
	-	LL (%)	Liquid limit	60	68-131	91.7	18.6
Kiran Jacob	Kuttanad (India) —	PL (%)	Plastic limit	60	29-69	46.2	16.1
(2016)		PI (%)	Plasticity index	60	27-62	45.6	23.1
	-	LI	Liquidity index	60	0.25-0.74	0.47	17.6
	-	Cc	Compression index	60	0.38-0.79	0.57	18.2
V. (1000)	Tseung Kwan O (Hong	PI (%)	Plasticity index	35	8.57-49.0	29.1	29.4
Y III (1999)	Kong)	Cc	Compression index	35	0.09-1.24	0.59	47.3
Zhu and Graham	Chek Lap Kok International Airport	Cc	Compression index	62	0.097-0.498	0.305	34.8
(2001)	(Hong Kong) –	C _{ur}	Swell index	62	0.009-0.060	0.031	35.6
	New Jersey Offshore (USA)	w (%)	Natural moisture content	14	49.0-61.5	53.5	7.0
		LL (%)	Liquid limit	14	61.7-88.4	72.1	9.4
Koutsoftas		PL (%)	Plastic limit	14	26.4-36.1	30.9	10.7
and Ladd		PI (%)	Plasticity index	14	32.7-52.3	41.2	11.9
(1985)		LI	Liquid index	14	0.47-0.63	0.55	11.1
	_	$s_u\!/\!\sigma'_v$	Normalized UC	14	0.82-1.42	1.02	15.2
	-	s_u/σ'_v	Normalized VST	12	0.89-1.20	1.03	10.3
		w (%)	Natural moisture content	10	43.6-68.9	55.8	17.4
	_	LL(%)	Liquid limit	10	28.4-44.3	35.9	17.0
_		PL (%)	Plastic limit	10	16.9-24.3	23.1	12.6
Tavenas et al. (1975)	Saint-Alban, Quebec (Canada)	PI (%)	Plasticity index	10	10.5-20.0	15.2	24.9
(1973)	(Culludu) _	LI	Liquid index	10	1.95-3.42	2.47	19.2
		s_u/σ'_v	Normalized UU	11	0.53-0.75	0.63	11.7
		s_u/σ'_v	Normalized VST	15	0.52-1.01	0.61	19.9
Clough and	Hamilton Air Force	w (%)	Natural moisture content	16	82.0-93.9	90.0	3.5

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Denby (1980)	Base (USA)	LL(%)	Liquid limit	16	81.2-96.0	88.3	4.9
		PL(%)	Plastic limit	16	36.5-46.9	40.0	6.8
		PI (%)	Plasticity index	16	42.6-52.1	48.3	6.8
		LI	Liquid index	16	0.89-1.22	1.04	9.3
		s_u/σ'_v	Normalized VST	16	0.23-0.44	0.34	18.9
		w (%)	Natural moisture content	19	28.1-49.9	42.2	16.4
		LL(%)	Liquid limit	19	49.8-98.5	81.4	13.3
		PL(%)	Plastic limit	19	16.9-31.4	26.1	13.8
		PI (%)	Plasticity index	19	32.9-68.6	55.3	15.6
Azzouz and	Empire Louisiana	LI	Liquid index	19	0.09-0.41	0.29	27.2
Lutz (1986)	(USA)	s_u/σ'_v	Normalized VST	13	0.11-0.25	0.19	23.6
	-	Bq	Pore pressure parameter	13	0.22-0.62	0.54	19.0
		s_u/σ'_v	Normalized UU	10	0.10-0.41	0.21	39.2
		s_u/σ'_v	Normalized UC	13	0.06-0.28	0.20	29.3
		\mathbf{q}_{tl}	Normalized cone tip resistance	13	5.15-6.44	5.75	10.3
		w (%)	Natural moisture content	12	30.7-40.4	34.9	7.7
		LL(%)	Liquid limit	10	19.3-38.6	34.6	8.0
Simons	Dramman (Norway)	PL(%)	Plastic limit	10	16.7-20.1	18.5	5.8
(1960)	Drammen (Norway)	PI (%)	Plasticity index	10	11.4-19.4	16.1	14.3
		LI	Liquid index	10	0.71-1.33	1.06	15.8
		s_u/σ'_v	Normalized VST	12	0.08-0.22	0.16	24.8
		w (%)	Natural moisture content	13	31.0-41.0	37.7	7.0
	_	LL(%)	Liquid limit	13	15.0-23.0	19.3	12.0
Finno (1989)	Evanston (USA)	PL (%)	Plastic limit	13	10.0-23.0	18.3	17.8
	(())) —	PI (%)	Plasticity index	14	17.0-28.0	22.8	10.2
	-	LI	Liquid index	13	0.06-0.35	0.24	47.6

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		$s_u\!/\sigma'_v$	Normalized VST	11	0.17-0.37	0.25	22.7
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	14	0.28-0.89	0.69	23.8
		St	Sensitivity	12	1.6-3.4	2.2	24.7
	-	s_u/σ'_v	Normalized UC	10	0.12-0.36	0.21	39.4
		q _{t1}	Normalized cone tip resistance	14	2.6-5.3	3.5	27.5
		w (%)	Natural moisture content	13	42.1-89.8	68.5	26.8
	_	LL(%)	Liquid limit	13	41.5-59.6	52.6	11.4
		PL(%)	Plastic limit	13	22.4-28.8	25.5	9.3
		PI (%)	Plasticity index	13	18.8-32.1	27.1	16.2
Konrad and	Gloucester, Ontario	LI	Liquid index	13	0.68-2.41	1.56	31.6
Law (1987a)	(Canada)	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	13	0.67-0.84	0.74	8.2
	-	OCR	Over consolidation ratio	13	1.43-2.16	1.68	11.2
		s_u/σ'_v	Normalized VST	11	0.42-0.64	0.51	15.7
		q _{t1}	Normalized cone tip resistance	13	5.2-7.3	6.2	9.7
		w (%)	Natural moisture content	13	25.8-64.1	43.3	25.6
		LL(%)	Liquid limit	13	39.0-87.7	60.8	26.9
		PL(%)	Plastic limit	13	17.1-32.5	23.4	18.6
		PI (%)	Plasticity index	13	21.0-65.1	37.4	35.2
Koutsoftas et	Hong Kong	LI	Liquid index	11	0.34-0.75	0.56	25.6
al. (1987)		s_u/σ'_v	Normalized VST	13	0.45-0.88	0.61	25.4
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	13	0.26-0.83	0.62	26.3
		OCR	Over consolidation ratio	12	1.9-3.8	2.7	23.8
		q_{t1}	Normalized cone tip resistance	13	5.8-16.9	10.5	35.7
D. 1.11 . 1		w (%)	Natural moisture content	12	61.9-85.3	75.0	7.7
Kochelle et al. (1988)	Louiseville, Quebec (Canada)	LL(%)	Liquid limit	12	61.5-71.1	64.8	5.2
× /		PL(%)	Plastic limit	12	25.2-28.4	26.3	4.0

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		PI (%)	Plasticity index	12	35.8-43.6	38.5	6.6
	-	LI	Liquid index	12	1.01-1.56	1.27	12.6
	-	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	11	0.18-0.71	0.60	24.9
	_	q _{t1}	Normalized cone tip resistance	12	10.2-16.6	13.2	17.0
		LI	Liquid index	13	0.54-0.92	0.74	17.2
~	Norfolk Road	s_u/σ'_v	Normalized UU	13	0.11-0.22	0.16	23.4
Chang (1991)	(Singapore)	$s_u\!/\!\sigma'_v$	Normalized VST	12	0.20-0.36	0.24	17.9
		q_{t1}	Normalized cone tip resistance	12	1.59-3.26	2.29	21.7
	Delta Amacuro –	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	14	0.36-0.60	0.51	16.1
Azzouz et al.	Offshore	s_u/σ'_v	Normalized VST	14	0.14-0.33	0.21	22.8
(1982)	(Venezuela)	q_{t1}	Normalized cone tip resistance	14	3.4-5.1	3.9	11.9
	– – Tucupita Offshore	w (%)	Natural moisture content	12	60.9-73.2	65.7	4.7
		LL(%)	Liquid limit	12	73.6-104.5	91.5	9.2
		PL(%)	Plastic limit	12	34.5-40.4	38.6	4.9
Azzouz et al.		PI (%)	Plasticity index	12	39.1-64.5	52.8	13.4
(1982)	(Venezuela)	LI	Liquid index	12	0.41-0.78	0.52	20.8
	_	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	13	0.39-0.52	0.45	9.3
	-	s_u/σ'_v	Normalized VST	10	0.18-0.38	0.24	22.2
	_	q_{t1}	Normalized cone tip resistance	13	3.5-6.2	4.4	15.3
		w (%)	Natural moisture content	17	43.1-102.7	67.1	28.4
		LL(%)	Liquid limit	15	27.5-52.0	39.8	19.4
Roy et al.	- Saint-Alban, Quebec	PL (%)	Plastic limit	15	14.8-25.2	21.0	14.8
(1982)	(Canada)	PI (%)	Plasticity index	10	11.8-20.7	16.2	20.1
	-	LI	Liquid index	15	0.96-3.32	2.25	27.1
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	17	0.27-0.34	0.31	7.4

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		OCR	Over consolidation ratio	14	2.18-2.43	2.31	3.7
	-	s_u/σ'_v	Normalized VST	14	0.53-0.63	0.56	6.0
	-	q_{t1}	Normalized cone tip resistance	14	8.72-10.64	9.65	6.0
		w (%)	Natural moisture content	17	39.4-79.5	55.2	19.4
		LL(%)	Liquid limit	17	61.4-437.8	83.6	25.4
		PL(%)	Plastic limit	17	19.6-40.0	28.0	22.2
		PI (%)	Plasticity index	17	37.5-116.6	55.5	37.6
Baligh et al.	East Atchafalaya Basin	LI	Liquid index	16	0.23-0.66	0.48	27.4
(1980)	(USA)	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	17	0.17-0.39	0.26	29.2
	_	OCR	Over consolidation ratio	17	1.11-1.94	1.32	14.5
	-	s_u/σ'_v	Normalized VST	18	0.31-0.44	0.39	8.5
	_	q_{t1}	Normalized cone tip resistance	17	3.05-4.10	3.76	9.0
	Troll East, North Sea (Norway) —	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	20	0.62-0.94	0.80	9.9
Amundsen et		$s_u\!/\sigma'_v$	Normalized VST	18	0.23-0.46	0.29	18.1
al. (1985)		q_{t1}	Normalized cone tip resistance	18	4.67-8.46	5.62	17.0
		LI	Liquid index	15	0.88-1.20	0.98	7.5
Battaglio et al. (1986)	Upplands-Vasby (Sweden)	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	15	0.83-1.07	0.99	6.5
	(2)	OCR	Over consolidation ratio	15	1.12-1.40	1.17	5.6
		w (%)	Natural moisture content	11	42.0-76.4	52.7	17.9
		LL(%)	Liquid limit	11	57.5-105.1	76.9	16.7
	_	PL (%)	Plastic limit	11	24.0-37.1	28.6	13.2
Wei et al.	Now Orleans (USA)	PI (%)	Plasticity index	11	30.6-68.0	48.3	21.0
(2010)	New Offeans (USA)	LI	Liquid index	11	0.30-0.61	0.49	18.4
	-	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	12	0.24-0.76	0.55	29.6
	-	s_u/σ'_v	Normalized UU	12	0.13-0.24	0.19	17.2
		q _{t1}	Normalized cone tip	12	1.92-4.00	2.52	22.0

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			resistance				
		w (%)	Natural moisture content	12	37.1-73.3	60.3	15.4
Chang (1991)	Kallang Basin (Singapore)	B_q	Pore pressure parameter	13	0.55-0.80	0.66	13.1
		\mathbf{q}_{t1}	Normalized cone tip resistance	13	1.54-2.40	2.04	12.4
		w (%)	Natural moisture content	13	30.0-47.0	41.4	12.5
		LL(%)	Liquid limit	13	32.1-51.1	43.9	12.1
Baligh et al.	Boston	PL(%)	Plastic limit	13	19.5-26.0	23.1	9.5
(1980)	(USA)	PI (%)	Plasticity index	13	12.6-28.7	20.9	22.0
		LI	Liquid index	13	0.59-1.48	0.89	27.4
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	12	0.13-0.73	0.56	31.3
Konrad and Law (1987b)	Saint-Marcel (Canada)	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	17	0.38-0.52	0.46	9.6
	Gullfaks A, North Sea (Norway)	w (%)	Natural moisture content	26	14.5-26.2	22.3	11.8
		LL(%)	Liquid limit	26	37.1-49.7	46.1	6.5
Lunne et al.		PL(%)	Plastic limit	26	16.3-21.3	19.7	5.8
(1985a)		PI (%)	Plasticity index	26	17.8-29.9	26.4	9.6
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	26	0.08-0.51	0.37	32.0
		q_{t1}	Normalized cone tip resistance	13	8.2-12.2	10.3	11.9
		w (%)	Natural moisture content	33	27.7-43.4	35.2	10.7
	Jamestown, Virginia —	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	19	0.34-0.67	0.55	12.8
Mayne (1991)	(USA)	OCR	Over consolidation ratio	19	1.72-3.50	2.78	17.3
		\mathbf{q}_{t1}	Normalized cone tip resistance	19	4.8-8.3	6.9	13.9
Senneset et al.		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	18	0.07-0.29	0.17	41.9
(1988)	Stjørdal (Norway)	q_{t1}	Normalized cone tip resistance	18	4.5-13.5	7.3	34.7
Mayne (2008)	Anchorage, Alaska	$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	22	0.43-0.91	0.68	20.4
wiayiic (2008)	(USA)	q _{t1}	Normalized cone tip	22	5.3-11.0	8.1	22.2

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			resistance				
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	18	0.68-0.86	0.78	7.3
Mayne (2008)	Brisbane (Australia)	\mathbf{q}_{t1}	Normalized cone tip resistance	18	2.4-3.3	2.9	9.0
Cadling and Odenstad (1950)	Gothenburg (Sweden)	St	Sensitivity	17	4.5-15.4	9.1	30.9
		w (%)	Natural moisture content	18	62.9-152.5	104.4	24.6
		LL(%)	Liquid limit	18	51.7-146.3	81.1	35.4
Ohtsubo et al.	Chinaichi Casa (Jaman)	PL(%)	Plastic limit	18	22.3-56.0	37.6	26.2
(1982)	Shiroishi, Saga (Japan) –	PI (%)	Plasticity index	18	22.3-96.5	43.5	47.0
	_	LI	Liquid index	18	0.89-2.62	1.70	30.6
	_	St	Sensitivity	14	13.4-101.2	38.6	63.4
	Hawkesbury, Ontario (Canada)	w (%)	Natural moisture content	13	61-90	79.4	10.4
		LL(%)	Liquid limit	13	53-72	64.3	8.1
Eden and		PL(%)	Plastic limit	13	25-28	26.4	2.9
Hamilton		PI (%)	Plasticity index	13	26-46	38.0	13.8
(1957)		LI	Liquid index	13	1.20-1.68	1.40	10.7
	_	St	Sensitivity	12	2.57-7.84	4.36	39.3
	-	s_u/σ'_v	Normalized VST	12	0.62-1.04	0.84	15.1
		w (%)	Natural moisture content	10	70.4-84.0	78.4	6.3
	_	LL(%)	Liquid limit	10	46.2-75.8	65.6	14.1
Eden and	National Research	PL(%)	Plastic limit	10	25.4-30.2	28.0	5.3
(1957)	(Canada)	PI (%)	Plasticity index	10	17.2-46.5	37.6	24.4
(1957)		St	Sensitivity	10	3.0-11.7	6.7	40.1
	-	s_u/σ'_v	Normalized VST	10	0.91-1.51	1.14	16.0
Andresen and		w (%)	Natural moisture content	17	26.4-43.6	35.7	12.6
Bjerrum 1957	Oiso (norway) –	LL(%)	Liquid limit	17	30.5-53.0	42.3	15.3

		PL(%)	Plastic limit	17	18.1-30.7	22.0	15.1
	-	PI (%)	Plasticity index	17	11.6-31.9	20.3	24.0
	-	LI	Liquid index	17	0.48-0.95	0.68	20.1
	-	St	Sensitivity	17	2.0-4.8	3.5	26.1
	-	Su	VST	15	21.9-45.3	33.2	19.6
		PL(%)	Plastic limit	15	33.78-76.5	52.2	22.2
Anderson (1982)	Rio de Janeiro (Brazil)	LI	Liquid index	15	1.09-1.63	1.32	11.6
(1) (1)	(21021)	St	Sensitivity	13	2.2-5.0	3.9	23.5
Parry (1968)	Launceston, Tasmania (Australia)	s_u/σ'_v	Normalized VST	17	0.25-0.77	0.64	7.6
		w (%)	Natural moisture content	20	15.5-54.1	44.1	12.7
	-	LL(%)	Liquid limit	19	27.3-57.0	45.3	11.7
	Gullfaks Location C, North Sea (Norway)	PL(%)	Plastic limit	19	15.4-20.9	19.3	4.5
Lunne et al		PI (%)	Plasticity index	15	23.8-36.3	28.7	12.6
(1985b)		LI	Liquid index	16	0.61-1.40	1.05	20.6
		$\mathbf{B}_{\mathbf{q}}$	Pore pressure parameter	40	0.45-0.90	0.71	16.1
		OCR	Over consolidation ratio	25	1.49-2.90	2.11	18.3
		q_{t1}	Normalized cone tip resistance	15	5.3-9.0	6.4	15.4
5 1 1		w (%)	Natural moisture content	24	22.4-46.5	34.0	20.7
Rutledge (1939)	Boston (USA)	C_{c}	Compression index	24	0.16-0.47	0.31	33.4
		C_{ur}	Swell index	24	0.03-0.11	0.07	36.4
		OCR	Over consolidation ratio	16	0.81-2.41	1.57	34.0
Briaud et al. (2007)	Houston (USA)	Cc	Compression index	17	0.09-0.48	0.31	40.6
		C_{ur}	Swell index	17	0.03-0.26	0.12	50.5
		w (%)	Natural moisture content	18	57.1-155.9	109.1	26.0
Ohtsubo et al. (2007)	Shiroishi, Saga (Japan)	LI	Liquid index	18	0.89-2.41	1.32	26.2
()	-	s_u/σ'_v	Normalized VST	18	0.26-1.15	0.51	36.4

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		St	Sensitivity	18	15-41	26.4	28.4
Bjerrum (1967)	Norway	s_u/σ'_v	Normalized VST	18	0.16-0.25	0.21	13.3
		w (%)	Natural moisture content	20	44.4-90.0	67.8	18.0
	_	LL(%)	Liquid limit	20	60.0-93.0	76.8	13.6
	_	PL(%)	Plastic limit	20	24.1-68.6	45.0	19.3
	The confluence of the	PI (%)	Plasticity index	20	17.6-54.0	31.8	32.7
Mayne and Frost (1988)	rivers, Washington,	OCR	Over consolidation ratio	22	1.37-3.00	1.87	23.0
	D.C. (USA)	SPT-N	Standard penetration test blow count	17	1.05-2.76	1.75	24.2
	_	E _{DMT} (MPa)	Modulus from DMT	21	3.11-8.78	4.85	33.0
	-	K _{DMT}	Lateral earth pressure coefficient from DMT	20	2.40-3.89	2.92	11.9
Marchetti	Montalto (Italy)	E _{DMT} (MPa)	Modulus from DMT	18	8.96-18.15	12.91	18.7
		K _{DMT}	Lateral earth pressure coefficient from DMT	18	3.13-4.35	3.55	9.2
(1980)	Porto Tolle (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	20	1.46-2.16	1.91	10.1
	Conca del Fuuno (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	25	2.65-3.30	2.96	6.2
Cheshomi and Ghodrati	Mashhad (Iran)	SPT-N	Standard penetration test blow count	15	8.97-50.33	27.33	45.1
(2014)		E _{PMT} (MPa)	Modulus from PMT	15	10.3-43.8	23.0	39.1
		w (%)	Natural moisture content	10	19.2-39.2	26.9	21.6
Privad (1007)	TAMU Riverside	LL(%)	Liquid limit	10	42.5-76.7	64.2	17.1
B11aud (1997)	Campus, Texas (USA)	PL(%)	Plastic limit	10	13.6-32.3	19.7	28.5
	-	PI (%)	Plasticity index	10	28.9-60.9	44.6	20.7
Skempton (1961)	Bradwell (UK)	\mathbf{K}_{0}	coefficient of earth pressure at rest	12	1.46-2.8	2.14	22.0
Watabe et al.		w (%)	Natural moisture content	16	32.1-43.7	37.0	9.5
(2003);Larsso n and	Drammen (Norway)	LL (%)	Liquid limit	16	18.2-22.1	20.2	5.5
Mulabdic	-	PL(%)	Plastic limit	16	34.8-48.1	40.5	10.2

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(1991)		PI (%)	Plasticity index	16	15.5-26.0	20.3	15.6
	_	K _{DMT}	Lateral earth pressure coefficient from DMT	16	2.96-4.52	3.85	16.2
		w (%)	Natural moisture content	18	16.5-68.7	59.5	10.3
		LL(%)	Liquid limit	18	22.9-25.8	24.7	3.4
Watabe et al		PL(%)	Plastic limit	18	52.4-71.7	61.6	9.3
(2003)	Pusan, Yangsan	PI (%)	Plasticity index	18	29.6-16.1	37.0	14.6
		OCR	Over consolidation ratio	10	1.15-1.31	1.22	4.3
	_	K _{DMT}	Lateral earth pressure coefficient from DMT	24	2.08-3.16	2.67	9.9
		OCR	Over consolidation ratio	16	1.81-2.21	1.96	6.2
		w (%)	Natural moisture content	24	51.0-99.9	83.9	16.3
Watabe et al.	 Yokohama port	LL(%)	Liquid limit	24	28.5-54.2	44.6	15.5
(2003);Tanaka et al. (2001a)	Yamashita (Japan)	PL(%)	Plastic limit	24	54.9-125.0	106.4	19.7
et al. (2001a)		PI (%)	Plasticity index	24	29.6-46.1	37.0	14.6
	-	K _{DMT}	Lateral earth pressure coefficient from DMT	26	2.10-3.11	2.57	7.6
		w (%)	Natural moisture content	13	64.0-87.2	73.8	9.1
		LL(%)	Liquid limit	13	61.2-72.1	66.6	4.7
Tanaka et al.	Louiseville Ouebec	PL(%)	Plastic limit	13	23.3-26.0	24.7	3.6
(2001a);Silves	(Canada)	PI (%)	Plasticity index	13	36.8-46.1	419	6.5
ui (2003)		LI	Liquid index	13	0.98-1.48	1.17	12.4
	_	K _{DMT}	Lateral earth pressure coefficient from DMT	13	5.07-11.95	7.28	31.7
		OCR	Over consolidation ratio	17	1.39-2.18	1.62	14.3
		w (%)	Natural moisture content	16	91.6-152.4	120.2	18.9
Kamei and Tanaka (2003)	Ariake Sea (Japan)	LL(%)	Liquid limit	16	68.0-128.3	104.2	16.7
		PL(%)	Plastic limit	16	31.5-52.2	44.1	12.3
	—	PI (%)	Plasticity index	16	36.5-80.6	60.2	20.8

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		LI	Liquid index	16	1.07-1.65	1.27	12.6
		s_u/σ'_v	Normalized VST	15	0.27-0.49	0.37	16.2
		K _{DMT}	Lateral earth pressure coefficient from DMT	17	1.73-3.36	2.12	21.6
	_	w (%)	Natural moisture content	10	47.7-98.0	74.5	20.6
		LL(%)	Liquid limit	10	61.6-109.3	87.4	17.4
		PL(%)	Plastic limit	10	22.6-35.8	26.6	16.4
Watabe et al	Ekachai Bangkok	PI (%)	Plasticity index	10	39.0-75.4	60.8	18.8
(2004)	(Thailand)	LI	Liquid index	10	0.61-0.90	0.78	12.8
		OCR	Over consolidation ratio	10	1.17-1.87	1.53	14.8
		E _{DMT} (MPa)	Modulus from DMT	13	1.02-2.46	1.12	33.9
		K _{DMT}	Lateral earth pressure coefficient from DMT	13	2.28-4.48	2.86	20.8
	– Merlion park (Singapore) –	w (%)	Natural moisture content	13	46.6-60.2	55.3	7.5
		PL(%)	Plastic limit	13	21.0-25.1	23.5	4.3
Tanaka et al.		LL(%)	Liquid limit	13	62.8-82.2	73.6	7.3
(2001b);Wata		PI (%)	Plasticity index	13	40.0-57.0	50.1	9.9
(2003)		LI	Liquid index	13	0.57-0.74	0.64	9.2
		s_u/σ'_v	Normalized VST	10	0.14-0.35	0.23	25.8
		K _{DMT}	Lateral earth pressure coefficient from DMT	15	2.38-3.94	2.95	14.9
		w (%)	Natural moisture content	10	58.3-66.5	61.6	4.8
		PL(%)	Plastic limit	10	29.1-36.5	33.4	7.6
		LL(%)	Liquid limit	10	53.7-69.0	59.1	8.5
Lunne et al.	Onsoy (Norway)	PI (%)	Plasticity index	10	19.2-32.5	25.9	15.7
(1990)	• • • •	LI	Liquid index	10	0.91-1.30	1.10	11.4
		\mathbf{K}_0	coefficient of earth pressure at rest	13	0.53-0.68	0.61	8.4
		s_u/σ'_v	Normalized VST	16	0.26-0.28	0.27	3.2

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		K _{DMT}	Lateral earth pressure coefficient from DMT	20	2.97-3.67	3.23	6.4
		w (%)	Natural moisture content	17	23.8-31.3	28.5	6.6
	_	PL(%)	Plastic limit	17	25.9-33.2	28.7	7.3
Lunne et al. (1990)	-	LL(%)	Liquid limit	17	67.4-81.2	73.9	4.8
	(UK)	PI (%)	Plasticity index	17	40.2-52.7	45.2	7.0
		K_0	coefficient of earth pressure at rest	10	1.07-1.73	1.49	17.3
		K _{DMT}	Lateral earth pressure coefficient from DMT	13	6.15-10.8	7.91	16.9
	_	w (%)	Natural moisture content	16	27.9-33.1	30.4	5.2
Hight et al Canons Park I or		PL(%)	Plastic limit	16	49.5-66.3	58.4	8.9
	Canons Park, London (UK)	LL(%)	Liquid limit	16	71.3-83.3	76.2	5.2
(2003)		PI (%)	Plasticity index	16	13.4-21.8	17.8	12.3
		E _{DMT} (MPa)	Modulus from DMT	10	18.29-27.82	22.8	11.1
		K _{DMT}	Lateral earth pressure coefficient from DMT	16	3.30-19.53	11.51	31.0
	Waterloo, London	w (%)	Natural moisture content	11	23.7-29.2	27.3	6.9
		PL(%)	Plastic limit	11	35.8-51.4	43.2	13.4
Hight et al.		LL(%)	Liquid limit	11	58.0-72.1	68.8	7.0
(2003)	$(\mathbf{U}\mathbf{K})$ –	PI (%)	Plasticity index	11	20.5-34.0	25.7	16.2
		\mathbf{K}_{0}	coefficient of earth pressure at rest	13	1.09-1.54	1.29	11.0
Takemura et	Can Tho site. Mekong	E _{DMT} (MPa)	Modulus from DMT	11	3.04-4.55	3.71	12.2
al. (2006)	Delta (Vietnam)	K _{DMT}	Lateral earth pressure coefficient from DMT	11	2.52-3.11	2.87	7.1
Coutinho at	_	E _{DMT} (MPa)	Modulus from DMT	16	2.41-3.82	3.01	13.7
al.	_	SPT-N		23	1-4	2.3	34.8
(2006);Coutin ho et al.	RRS1, Recife (Brazil)	M _d (MPa)	Constrained tangent modulus	13	2.38-4.45	3.21	20.8
(2008)	-	K _{DMT}	Lateral earth pressure coefficient from DMT	23	2.59-5.28	3.61	25.8

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		OCR	Over consolidation ratio	11	0.84-1.04	0.94	7.8
	– RRS2, Recife (Brazil)	M _d (MPa)	Constrained tangent modulus	11	0.31-0.86	0.49	37.2
	_	K _{DMT}	Lateral earth pressure coefficient from DMT	17	2.28-3.66	2.71	12.0
		w (%)	Natural moisture content	17	56.1-72.2	63.1	7.1
	_	LL(%)	Liquid limit	17	39.8-55.5	50.8	7.2
		PL(%)	Plastic limit	17	24.3-33.5	30.4	7.2
DeCreation d	_	PI (%)	Plasticity index	17	12.1-23.9	20.4	15.6
Lutenegger	CVVC, New England	LI	Liquid index	17	1.23-2.91	1.65	24.4
(2002)	(USA) –	E _{DMT} (MPa)	Modulus from DMT	13	4.7-3.4	5.0	4.6
	_	St	Sensitivity	19	4.6-7.6	6.11	12.4
	-	s_u/σ'_v	Normalized VST	17	4.6-7.5	0.06	20.5
		K _{DMT}	Lateral earth pressure coefficient from DMT	17	2.45-5.25	3.59	23.9
Leong and Rahardjo (2002)	Jurong Formation - residual soil (Singapore)	SPT-N	Standard penetration test blow count	14	6.8-29.2	20.1	36.0
		w (%)	Natural moisture content	12	30.5-40.0	35.6	10.2
Bihs et al		PI (%)	Plasticity index	12	3.2-9.0	6.2	35.2
(2013)	Kvennild (Norway)	OCR	Over consolidation ratio	11	1.45-2.62	1.09	17.4
	_	M _d (MPa)	Constrained tangent modulus	10	2.32-5.51	4.11	26.4
		w (%)	Natural moisture content	13	19.9-23.5	21.9	6.1
	_	PL(%)	Plastic limit	13	17.9-21.2	20.1	5.9
Cabrera et al. (2013)	tunnel next Barcelona (Spain)	LL(%)	Liquid limit	13	26.3-31.8	29.7	5.1
(2013)	(5)	PI (%)	Plasticity index	13	7.1-11.4	9.6	11.6
	-	E _{PMT} (MPa)	Modulus from PMT	13	15.0-26.9	22.1	19.8
Tong et al.	Nanjing Fourth Bridge	w (%)	Natural moisture content	18	299.1-46.6	34.3	14.1
(2013)	site A, China	PL(%)	Plastic limit	18	16.5-25.2	20.8	12.6

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		LL(%)	Liquid limit	18	27.4-39.4	33.4	13.1
	-	PI (%)	Plasticity index	18	9.2-16.4	12.6	18.8
	_	SPT-N	Standard penetration test blow count	12	6.3-1.0	7.9	15.9
Wang et al. (2013)	Tsengwen reservoir (Taiwan)	E _{DMT} (MPa)	Modulus from DMT	10	1.47-2.43	1.80	14.6
	N1, north side of Tevere river (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	24	1.35-3.90	1.96	18.1
	N2 north side of	E _{DMT} (MPa)	Modulus from DMT	18	3.90-11.48	8.40	28.0
Bosco and	Tevere river (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	18	1.57-2.51	2.13	14.9
(2016)	N3 north side of	E _{DMT} (MPa)	Modulus from DMT	24	2.38-14.74	8.23	43.9
	Tevere river (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	24	1.54-2.96	2.28	13.8
	South side of Tevere river (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	16	1.14-3.01	2.12	23.1
Cao et al	Bradford West	E _{DMT} (MPa)	Modulus from DMT	23	1.34-8.59	5.49	27.6
(2016)	Gwillimbury, Ontario (Canada)	K _{DMT}	Lateral earth pressure coefficient from DMT	21	3.09-6.64	5.00	21.3
Bihs et al.	Limeniale (Instand)	M _d (MPa)	Constrained tangent modulus	10	0.48-1.52	0.89	46.8
(2010)	Limenck (neiand)	K _{DMT}	Lateral earth pressure coefficient from DMT	10	2.51-5.14	3.37	29.1
	200th Street Overpass	E _{DMT} (MPa)	Modulus from DMT	11	0.99-2.10	1.54	24.2
	test, Vancouver (Canada)	K _{DMT}	Lateral earth pressure coefficient from DMT	10	2.38-3.45	2.82	13.6
		w (%)	Natural moisture content	10	38.4-49.9	42.5	8.5
Cruz (2009)		PL(%)	Plastic limit	10	22.9-28.3	26.4	6.0
	Surrey, Vancouver	LL(%)	Liquid limit	10	34.4-48.4	40.7	10.5
	(Canada)	E _{DMT} (MPa)	Modulus from DMT	12	0.24-1.09	0.71	35.3
		K _{DMT}	Lateral earth pressure coefficient from DMT	13	2.12-2.80	2.40	8.0
Iwasaki et al.	Komatsugawa, Tokyo	OCR	Over consolidation ratio	10	1.42-2.22	1.75	16.2

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(1991)	Bay (Japan)	K _{DMT}	Lateral earth pressure coefficient from DMT	12	3.91-4.01	3.30	11.0
Windle and	Hendon, London (UK)	K_{0}	coefficient of earth pressure at rest	10	2.20-3.60	2.88	17.2
Wroth (1977)	, , ,	E _{PMT} (MPa)	Modulus from PMT	10	38.0-90.6	66.2	22.0
		OCR	Over consolidation ratio	11	1.15-1.19	1.17	1.2
Larsson and	Lilla Mellosa,	E _{DMT} (MPa)	Modulus from DMT	13	0.60-2.01	1.10	39.4
	Stockholm (Sweden)	K _{DMT}	Lateral earth pressure coefficient from DMT	12	2.96-5.19	3.71	16.1
Eskilson (1989)	Ska-Edeby, Stockholm (Sweden)	K _{DMT}	Lateral earth pressure coefficient from DMT	11	3.11-5.57	3.82	22.1
		E _{DMT} (MPa)	Modulus from DMT	13	0.50-1.79	1.08	34.8
	Norrkoping (Sweden)	K _{DMT}	Lateral earth pressure coefficient from DMT	13	2.08-3.65	3.04	16.7
Robertson et	Langley site, BC	E _{DMT} (MPa)	Modulus from DMT	14	1.16-3.98	1.93	29.9
al. (1988)	(Canada)	K _{DMT}	Lateral earth pressure coefficient from DMT	16	5.21-7.57	6.08	11.2
Roque et al.	Glava, Stjordal	E _{DMT} (MPa)	Modulus from DMT	15	1.52-3.00	2.32	20.7
(1988)	(Norway)	K _{DMT}	Lateral earth pressure coefficient from DMT	18	3.58-8.79	4.98	28.3
Carrassco et al. (2004)	Madrid (Spain)	SPT-N	Standard penetration test blow count	19	39.3-78.4	55.9	19.3
Carrassco et	Madrid (Spain)	SPT-N	Standard penetration test blow count	40	42.3-95.9	75.3	15.9
al. (2004)		E _{PMT} (MPa)	Modulus from PMT	22	68.3-271.0	160.6	36.1
Sandven et al.		OCR	Over consolidation ratio	14	2.25-5.07	3.15	26.4
(2004)	Trondheim (Norway)	M _d (MPa)	Constrained tangent modulus	13	2-8	4.6	41.9
Cavallaro et	Catania STM M6	E _{DMT} (MPa)	Modulus from DMT	19	2.58-7.05	4.63	23.2
al. (2006)	(Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	19	2.03-3.41	2.47	16.5
Cavallaro et	Bellini Garden. Catania	E _{DMT} (MPa)	Modulus from DMT	19	16.67-24.37	19.32	11.1
al. (2016)	(Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	18	5.62-8.00	6.85	9.3

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Totani et al.	Colle Cretone Pineto	E _{DMT} (MPa)	Modulus from DMT	19	27.71-28.58	33.72	10.1
(2016)	(Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	19	6.36-8.54	7.16	8.2
	_	w (%)	Natural moisture content	17	39.5-52.0	46.1	7.5
		PL(%)	Plastic limit	19	26.0-39.1	32.2	10.7
	_	LL(%)	Liquid limit	19	53.0-84.5	69.8	13.5
Totani et al.	Garigliano river, (Italy)	PI (%)	Plasticity index	19	22.2-49.9	37.6	18.8
(1998)	_	E _{DMT} (MPa)	Modulus from DMT	21	9.21-15.19	11.40	17.2
	_	K _{DMT}	Lateral earth pressure coefficient from DMT	21	3.35-5.58	4.67	17.5
	Santa Barbara (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	48	1.49-3.11	2.03	20.2
		w (%)	Natural moisture content	15	17.6-27.2	22.0	12.8
	Port Huron, Michigan (USA) –	PL(%)	Plastic limit	15	15.1-21.3	17.3	12.7
Chen and		LL(%)	Liquid limit	15	26.0-41.2	32.7	18.4
Mayne (1994)		E _{DMT} (MPa)	Modulus from DMT	26	4.24-10.72	6.38	28.2
		K _{DMT}	Lateral earth pressure coefficient from DMT	16	3.50-4.50	3.94	8.5
		w (%)	Natural moisture content	14	29.3-38.3	32.4	8.3
		PL(%)	Plastic limit	14	16.5-21.6	18.3	8.9
Rouainia et al.	Western Avenue in	LL(%)	Liquid limit	14	32.1-44.8	37.9	9.6
(2017)	Aliston, Boston (USA) –	PI (%)	Plasticity index	14	14.4-24.7	19.6	16.4
	_	\mathbf{K}_0	coefficient of earth pressure at rest	18	0.52-1.12	0.81	20.8
		OCR	Over consolidation ratio	10	1.31-2.14	1.78	12.8
Kelly et al.	Ballina, New South	$\mathbf{S}_{\mathbf{t}}$	Sensitivity	13	2.3-3.8	2.9	14.9
(2017)	wales (Australia) –	K _{DMT}	Lateral earth pressure coefficient from DMT	24	1.68-9.95	4.16	49.4
White et al.	IA Highway 191	E _{DMT} (MPa)	Modulus from DMT	12	0.51-2.59	1.40	45.8
White et al. (2007)	IA Highway 191 (Neola, Iowa) (USA)	K _{DMT}	Lateral earth pressure coefficient from DMT	12	0.82-2.08	1.34	28.0

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Santagata et		\mathbf{K}_0	coefficient of earth pressure at rest	17	0.46-0.50	0.48	2.4
al. (2005)		w (%)	Natural moisture content	17	33.4-39.0	35.9	5.0
	_	LI	Liquid index	17	0.43-0.68	0.55	14.3
Kuo (1994)	National Taiwan University (Taiwan)	K _{DMT}	Lateral earth pressure coefficient from DMT	12	2.35-5.41	3.58	25.5
Powell and Uglow (1988)	Grangemouth	K _{DMT}	Lateral earth pressure coefficient from DMT	15	2.83-4.29	3.33	12.2
Penna (2006)	Alemoa-Santos, Sao Paulo (Brazil)	K _{DMT}	Lateral earth pressure coefficient from DMT	23	0.85-2.11	1.87	14.3
Sabatini et al. (2002)	Connecticut River Valley, Massachusetts (USA)	K _{DMT}	Lateral earth pressure coefficient from DMT	17	2.59-6.80	4.06	30.0
Huang et al. (2001)	sugarcane field beside high-speed rail (Taiwan)	K _{DMT}	Lateral earth pressure coefficient from DMT	10	2.03-3.99	2.90	23.7
	Venezia Lido (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	23	1.42-2.80	1.91	16.9
Marchetti et al. (2001)	Stagno Livorno (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	23	2.06-3.89	2.66	15.8
	S. Barbara (Italy)	K _{DMT}	Lateral earth pressure coefficient from DMT	11	12.16-19.84	15.12	13.7

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 Table 1.A2. Site-specific sand property statistics (databases: SAND/7/2794 & SAND/13/4113)

Source	Site Description]	Property	No. of tests (≥10)	Range of data	Property mean	Property COV (%)
Lee and Seed (1967)	Sacramento River Sand	φ (°)	Friction angle	39	30.9-40.9	34.2	7.7
Kulhawy and Mayne (1990)	Ticino	φ (°)	Friction angle	64	32.6-47.5	52.0	9.3
Chin et al. (1988)	Hsinta Power Plant, Kaohsiung, Taiwan —	q_{c1n}	Normalized cone tip resistance	35	17.6-106.8	62.4	31.2
		$(N_1)_{60}$	Normalized SPT-N	35	6.2-33.3	19.1	35.0
Huang et al. (1999)	Mia-Liao, Taiwan	q _{c1n}	Normalized cone tip resistance	40	40.4-245.2	142.4	34.1

Huang (1991)	Washed mortar sand	q _{c1n}	Normalized cone tip resistance	42	15.3-406.7	160.8	71.7
Bozbey and	Istonbul (Tuelcov)	SPT-N	Corrected SPT-N	53	15.1-70.5	45.9	31.3
Togrol (2010)	Istanbul (Turkey) –	E _{PMT} (MPa)	Modulus from PMT	53	11.7-39.6	26.1	29.2
	Ching Cheung	w (%)	Natural moisture content	31	14.7-45.8	24.1	27
	Road landslide in	$ ho_b$ (kg/m ³)	Bulk density	31	1.7-2.1	2.0	4.9
GEO (1989)	(Completely	$ ho_d$ (kg/m ³)	Dry density	31	1.1-1.9	1.6	9.5
	decomposed	c (kPa)	Cohesion	30	1.7-39.6	17.5	51.4
	granne) –	φ (°)	Friction angle	30	29.2-45.6	37.0	9.6
		$\gamma_{\rm d}({\rm kN/m^2})$	Dry unit weight	14	15.7-17.0	16.4	2.6
	Erksak (Man-made) —	e	Initial void ratio	14	0.53-0.66	0.59	7.2
Been et al. (1987)		D _r (%)	Initial relative density	14	69.2-98.9	86.3	11.1
		q _{c1n}	Corrected normalized qc	14	30.2-265.6	146.7	45.8
	Hokksund	Φ (°)	Friction angle	14	31.4-40.2	36.1	7.8
Parkin et al.		q _{c1n}	Corrected normalized qc	28	68.5-430.9	254.6	40.1
(1980)		$\Phi\left(^{\circ} ight)$	Friction angle	28	38.5-49.8	45.7	5.3
Houlsby and	Leighton Buzzard	q_{c1n}	Corrected normalized qc	19	26.2-484.3	196.7	68.0
Hitchman (1988)		$\Phi\left(^{\circ} ight)$	Friction angle	19	33.2-46.4	39.5	11.5
Villet and	Lone Star 60#	q_{c1n}	Corrected normalized qc	16	46.2-171.9	101.4	39.8
Mitchell (1981)		$\Phi\left(^{\circ} ight)$	Friction angle	16	35.5-45.4	39.9	8.7
	Monterey 0	qc1n	Corrected normalized qc	15	73.0-199.4	144.4	27.2
	•	$\Phi\left(^{\circ} ight)$	Friction angle	16	36.1-40.7	38.1	4.2
Greeuw et al.	Oostershelde	q_{c1n}	Corrected normalized qc	10	63.6-261.4	131.2	55.1
(1988)		Φ (°)	Friction angle	10	35.5-43.2	39.0	7.2
Unknown	Toyoura	q _{c1n}	Corrected normalized	11	27.6-297.0	116.8	68.0

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			qc				
	_	Φ (°)	Friction angle	11	34.6-44.1	39.1	7.6
		e	Initial void ratio	17	0.54-0.70	0.63	7.0
	A Ollawa —	φ (°)	Friction angle	17	20.1-36.5	32.3	5.9
	P. Ottowo	e	Initial void ratio	13	0.48-0.66	0.60	10.9
	B Ollawa –	φ (°)	Friction angle	13	32.5-40.8	35.7	7.9
Salgado et al.	C Ottowo	e	Initial void ratio	12	0.42-0.58	0.53	11.3
(2000)	C Ollawa –	φ (°)	Friction angle	12	33.7-41.3	37.0	6.2
	D Ottowa	e	Initial void ratio	17	0.32-0.61	0.47	19.9
	D Ollawa –	φ (°)	Friction angle	17	32.4-45.5	38.3	12.5
	F Ottawa	e	Initial void ratio	11	0.38-0.54	0.47	10.4
	L'Ottawa	φ (°)	Friction angle	11	34.3-38.8	35.9	4.8
Huong (1001)	Messina —	D ₅₀ (mm)	Median grain size	25	2.18-3.67	2.80	14.3
Huang (1991)	Messilia	$(N_1)_{60}$	Normalized SPT-N	25	21.2-39.5	28.6	16.5
Ghionna and	Holocene Coastal Plain	D ₅₀ (mm)	Median grain size	22	1.45-4.96	3.00	32.0
(1991)		$(N_1)_{60}$	Normalized SPT-N	25	14.4-31.9	22.1	25.6
Chapman and Donald (1981)	Frankston	φ (°)	Friction angle	11	35.2-41.4	39.6	5.0
Huntsman et al. (1986)	Caisson-retained Island, Canadian Beaufort Sea (Canada)	\mathbf{K}_0	coefficient of earth pressure at rest	15	0.75-1.99	1.09	35.6
Unknown	Mantova (Italy) -	\mathbf{K}_0	coefficient of earth pressure at rest	15	0.32-1.23	0.64	34.0
UIKIIOWII	Mantova (Itary)	q_{c1n}	Normalized cone tip resistance	15	66.9-148.9	94.4	24.3
		$(N_1)_{60}$	Normalized SPT-N	17	5.9-19.1	13.1	30.6
Marchetti (1980)	Torre Oglio (Italy)	q _{c1n}	Normalized cone tip resistance	22	13.1-145.7	72.1	45.2
Marchetti et al. (2001)	Chieti (Italy)	E _{DMT} (MPa)	Modulus from DMT	13	10.7-35.3	20.8	38.9

Yagiz et al. (2008)	Denizli (Turkey)	E _{PMT} (MPa)	Modulus from PMT	10	8.2-15.4	10.6	29.0
Kuo (1994)	Wugu Industrial Area (Taiwan)	q _{c1n}	Normalized cone tip resistance	11	27.8-74.9	46.9	32.0
	Po River Valley	E _{DMT} (MPa)	Modulus from DMT	25	30.0-105.0	60.0	32.5
Marchetti (1985)	(Italy)	q _{c1n}	Normalized cone tip resistance	25	66.0-220.9	120.0	34.6
Sandven (2003)	Halen (Norway)	E _{DMT} (MPa)	Modulus from DMT	10	5.14-8.32	6.36	16.0
	Nanjing Fourth	$(N_1)_{60}$	Normalized SPT-N	29	4.3-11.7	5.7	36.3
Tong et al. (2012)	Bridge site A (China)	SPT-N	Standard penetration test blow count	24	4.3-11.0	7.3	18.4
Mlvnarek et al	Cylindrical tank	E _{DMT} (MPa)	Modulus from DMT	11	30.2-84.5	56.0	27.7
(2012)	(Poland)	q _{c1n}	Normalized cone tip resistance	12	62.8-297.5	161.9	49.9
Tschuschke et al. (2013)	Zelazny Most dump (Poland)	q_{c1n}	Normalized cone tip resistance	28	19.9-101.3	55.0	33.4
	N1, north side of Tevere river (Italy)	q_{c1n}	Normalized cone tip resistance	18	6.1-57.2	28.2	61.1
Bosco and Monaco (2016)	N3, north side of Tevere river (Italy)	q_{c1n}	Normalized cone tip resistance	11	8.0-30.2	15.2	43.0
	south side of Tevere river (Italy)	q_{c1n}	Normalized cone tip resistance	14	6.3-25.1	14.1	38.5
	Fraser River Delta	$(N_1)_{60}$	Normalized SPT-N	16	13.6-34.9	21.9	32.2
	in Richmond, — Vancouver (Canada)	q _{c1n}	Normalized cone tip resistance	12	56.7-140.3	79.9	30.5
Cruz (2009)	Massey Tunnel, Richmond, Vancouver (Canada)	qcin	Normalized cone tip resistance	15	41.1-77.0	54.1	18.0
	Patterson Park, Delta, Vancouver (Canada)	q _{c1n}	Normalized cone tip resistance	10	83.3-122.6	100.5	15.0
Giacheti et al. (2006);Mio and	Bauru, Sao Paulo (Brazil)	q _{c1n}	Normalized cone tip resistance	15	37.0-53.5	44.6	11.5
Giacheti (2004)	Sao Carlos, Sao	$(N_1)_{60}$	Normalized SPT-N	18	3.4-13.5	8.9	36.7

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	Paulo (Brazil)	qcin	Normalized cone tip resistance	12	11.5-37.6	19.4	36.1
Foa et al. (2004)	Salvador, Bahia state (Brazil)	(N ₁) ₆₀	Normalized SPT-N	11	4.0-15.0	8.4	38.4
Carrassco et al. (2006)	Madrid (Spain)	SPT-N	Standard penetration test blow count	22	40.5-99.0	69.4	23.8
	Cajamar, Sao Paulo (Brazil)	E _{DMT} (MPa)	Modulus from DMT	19	20.2-62.1	32.6	29.6
Penna (2006)	Embu, Sao Paulo	$(N_1)_{60}$	Normalized SPT-N	17	1.8-14.3	9.7	38.8
	(Brazil)	E _{DMT} (MPa)	Modulus from DMT	14	8.9-33.4	22.6	32.8
Anderson et al.	Statesville, Iredell	N_{60}	Corrected SPT-N	12	5.2-10.0	6.8	20.6
(2006)	County, North — Carolina (USA)	E _{DMT} (MPa)	Modulus from DMT	12	8.75-13.17	11.72	10.9
Maugeri and Monaco (2006)	San Giuseppe La Rena, Catania (Italy)	E _{DMT} (MPa)	Modulus from DMT	18	37.9-98.8	71.4	24.6
Rocha et al.	UNESP research	E _{DMT} (MPa)	Modulus from DMT	20	27.1-48.8	36.5	18.1
(2016) site, Bauru, Sao Paulo (Brazil)		E _{PMT} (MPa)	Modulus from PMT	14	19.5-32.8	15.6	15.7
da Fonesca and Coutinho (2008)	Subway Station, Sao Paulo (Brazil)	K_0	coefficient of earth pressure at rest	13	1.0-3.8	2.2	36.9
Akbar et al. (2008)	River Ravi, Lahore (Pakistan)	N_{60}	Corrected SPT-N	10	9.6-21.5	14.9	27.1
Cao et al. (2008)	Changi East reclamation site (Singapore)	E _{DMT} (MPa)	Modulus from DMT	12	23.3-53.5	36.2	30.0
Cao et al. (2008)	Changi East reclamation site (Singapore)	E _{DMT} (MPa)	Modulus from DMT	10	27.6-46.4	35.7	17.9
		E _{DMT} (MPa)	Modulus from DMT	11	39.1-59.5	51.5	10.4
Arroyo et al.	Rampa1 (fine sand)	q_{c1n}	Normalized cone tip resistance	11	53.5-103.1	71.4	19.5
(2008)	—	E _{DMT} (MPa)	Modulus from DMT	11	9.3-57.2	38.6	45.7
	Rampal (silt)	q c1n	Normalized cone tip resistance	11	10.9-53.4	26.1	46.5
Totani et al.	Parma in Po River	E _{DMT} (MPa)	Modulus from DMT	13	8.0-25.8	14.0	36.8

(1998)	plain (Italy) (clay) Parma in Po River plain (Italy) (silt)	E _{DMT} (MPa)	Modulus from DMT	12	5.2-30.3	15.0	39.3
Brahana and	Atlanta Olympic	N_{60}	Corrected SPT-N	12	7-18	13.4	28.2
Wang (1998)	Stadium (USA)	E _{DMT} (MPa)	Modulus from DMT	11	5.68-13.47	10.28	25.6
Sabatini et al.	i et al. Piedmont Province, q _{c1n}		Normalized cone tip resistance	11	18.9-36.1	27.7	17.7
(2002) Alabama (Alabama (USA)	E _{DMT} (MPa)	Modulus from DMT	10	1.47-2.83	2.21	19.4
Di Mariano et al. (2019)	Verge de Montserrat Station, Llobregat River delta, Bercelona (Spain)	E _{DMT} (MPa)	Modulus from DMT	12	10.9-42.0	26.9	37.2
Lutenegger (1986)	Russe (Bulgaria)	E _{DMT} (MPa)	Modulus from DMT	12	5.63-26.63	13.40	46.4
Huang et al. (2001)	sugarcane field beside high-speed rail (Taiwan)	E _{DMT} (MPa)	Modulus from DMT	12	7.13-27.34	15.61	41.6
Marchetti (1991)	Po river (Italy)	E _{DMT} (MPa)	Modulus from DMT	15	25.26-52.00	34.93	20.3
Modoni and	Boiszowy Nowe	E _{DMT} (MPa)	Modulus from DMT	15	26.7-78.2	49.9	32.5
Bzówka (2012)	(Poland)	q_{c1n}	Normalized cone tip resistance	14	81.7-185.3	135.8	21.5
Ku and Mayne (2015)	San Matteo (Italy)	K ₀	coefficient of earth pressure at rest	15	0.43-1.15	0.71	25.8

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 Table 1.A3. Site-specific intact rock property statistics (databases: ROCK/13 & ROCK/9/4069)

Source	Site Description	Property		No. of tests (≥30)	Range of data	Property mean	Property COV (%)
Diamantis et al. (2009)	Western Othrys mt., Greece,	σ _{ci} (MPa)	Uniaxial compressive strength	32	19.2-125.7	60.3	44.8
	(Serpentinite)	I _{s50} (MPa)	Point load index	32	1.0-4.9	3.1	35.1

		n (%)	Porosity	32	0.4-4.6	1.5	70.6
	-	γ (kN/m ³)	Unit weight	32	24.4-26.7	25.6	2.2
		σ _{ci} (MPa)	Uniaxial compressive strength	63	6.3-107.5	43.1	52.8
	Metsovo road tunnel, Epirus,	I _{s50} (MPa)	Point load index	63	0.3-4.5	1.36	69.85
	Greece (Gabbro)	E _i (GPa)	Young's modulus	63	1.0-9.8	4.5	50.9
Aggistalis et al.		R _L	Schmidt hammer rebound number	63	19.5-57.2	32.2	25.0
(1996)	Metsovo road tunnel, Epirus, Northern Greece (Basalt)	σ _{ci} (MPa)	Uniaxial compressive strength	30	17.1-91.2	46.7	41.0
		I _{s50} (MPa)	Point load index	30	0.7-3.4	3.1	26.5
		E _i (GPa)	Young's modulus	30	1.2-12.1	5.1	51.8
		R _L	Schmidt hammer rebound number	30	21.8 - 55.0	42.4	16.1
	Siwalik Hills, central Nepal (Sandstone)	σ _{ci} (MPa)	Uniaxial compressive strength	44	1.3-51.6	26.8	52.2
Tamrakar et al.		Is50 (MPa)	Point load index	44	0.1-4.0	1.3	75.5
(2007)		E _i (GPa)	Young's modulus	44	0.1-1.1	0.8	32.3
		R _L	Schmidt hammer rebound number	44	12-53	31.6	32.5
Zorlu et al. (2008)	Karakaya, Greece (Sandstone)	σ _{ci} (MPa)	Uniaxial compressive strength	61	17.5-107.8	57.9	43.7
		σ _{ci} (MPa)	Uniaxial compressive strength	43	35-52	43.4	12.6
Arman et al.	Kondina Turkay (Limastana)	I _{s50} (MPa)	Point load index	43	3.0-4.0	3.7	11.9
(2007)	Kanuna, Turkey (Liniestone) –	σ _{bt} (MPa)	Brazilian tensile strength	43	4.0 -16.0	10.7	21.0
	_	R _L	Schmidt hammer rebound number	43	33-40	36.0	5.7
		σ _{ci} (MPa)	Uniaxial compressive strength	39	8.1-35.6	21.6	32.6
Yilmaz and	– Sivas, Turkev (Gypsum)	I _{s50} (MPa)	Point load index	39	1.2-3.2	2.5	20.6
Yuksek (2008)		E _i (GPa)	Young's modulus	39	15.7-42.8	28.0	26.8
	-	R _L	Schmidt hammer	39	27-48	37.1	14.9

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			rebound number				
		σ _{ci} (MPa)	Uniaxial compressive strength	40	6.3-196.5	64.2	86.5
		ρ (g/cm ³)	Density	40	2.1-2.7	2.5	5.6
Aydin and Basu (2005)	Hong Kong (Granite)	E _i (GPa)	Young's modulus	40	4.5-53.2	22.6	17.0
(2000)		n (%)	Porosity	40	1.3-21.0	9.8	61.8
	-	R _L	Schmidt hammer hardness	40	20-65.8	47.9	26.2
		σ _{ci} (MPa)	Uniaxial compressive strength	114	9.8-130.2	52.6	45.4
		I _{s50} (MPa)	Point load index	149	1.2-15.1	6.3	45.0
Kochay and Kilic		γ (kN/m ³)	Unit weight	172	22.3-28.3	25.7	4.0
(2006)	Turkey (Basalt)	E _i (GPa)	Young's modulus	99	8.9-89.1	39.3	43.8
		n (%)	Porosity	172	2.1-14.9	7.2	35.7
		V	Poisson ratio	172	0.2-0.4	0.3	9.7
		Gs	Specific gravity	172	2.8-3.0	2.9	1.7
Bastola and Chugh (2015)	Illinois (USA)	γ (kN/m ³)	Unit weight	44	17.5-25.9	22.8	8.1
		$\gamma (kN/m^3)$	Unit weight	66	16.3-20.1	18.2	4.5
Nefeslioglu (2013)	Firuzköy, Istanbul (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	66	0.7-4.1	1.9	40.1
		E _i (GPa)	Young's Modulus	65	0.03-0.44	0.13	60.0
		γ (kN/m ³)	Unit weight	40	20.5-25.7	23.4	5.4
Azimian and Ajalloeian (2014)	Shiraz, Fars Province (Iran)	σ _{ci} (MPa)	Uniaxial compression strength	40	15.3-88.9	47.8	41.1
		E _i (GPa)	Young's Modulus	40	5.6-30.7	15.9	41.2
		γ (kN/m ³)	Unit weight	40	25.4-27.1	26.3	1.5
Khanlari and	Hamadan Province (Iran)	I _{s50} (MPa)	Point load index	40	1.2-5.5	3.7	27.8
Addilor (2011)		σ _{ci} (MPa)	Uniaxial compression strength	40	56.2-104.0	87.1	14.0
Diamantis et al. (2011)	Greece	R _L	Schmidt hammer hardness	35	59-65	62.6	3.0
			61				

		σ _{ci} (MPa)	Uniaxial compression strength	35	65.2-241.6	142.1	32.2
		E _i (GPa)	Young's Modulus	35	26.4-69.3	44.6	30.7
	– Haji Mine, Bamyan Province (Afghanistan)	R _L	Schmidt hammer hardness	30	25.6-30.5	27.9	5.1
Dehghan et al.		I _{s50} (MPa)	Point load index	30	2.3-4.0	3.2	13.2
(2010)		σ _{ci} (MPa)	Uniaxial compression strength	30	22.7-71.5	39.0	32.3
		E _i (GPa)	Young's Modulus	30	3.0-11.5	5.4	37.1
	_	R _L	Schmidt hammer hardness	44	12-53	32.1	31.7
Tamrakar et al.	Simplify Hills (Nonal)	I _{s50} (MPa)	Point load index	44	0.05-4.00	1.27	75.5
(2007)	Siwalik Hills (Nepal)	σ _{ci} (MPa)	Uniaxial compression strength	44	1.3-51.6	26.7	52.2
		E _i (GPa)	Young's Modulus	44	0.06-1.09	0.75	32.3
	Cotai,Taipa,Coloane,Macau Peninsula (Macao)	R _L	Schmidt hammer hardness	145	16.8-56.5	43.7	17.1
Ng et al. (2015)		I _{s50} (MPa)	Point load index	145	1.08-8.56	4.85	36.0
		σ _{ci} (MPa)	Uniaxial compression strength	145	20.3-112.9	53.9	34.3
		I _{s50} (MPa)	Point load index	30	0.67-3.43	2.07	39.7
Aggistalis et al. (1980)	Epirus (Greece)	σ _{ci} (MPa)	Uniaxial compression strength	30	17.1-91.2	46.7	41.0
		E _i (GPa)	Young's Modulus	30	1.20-12.07	5.05	51.8
Endait and Iuneia		I _{s50} (MPa)	Point load index	41	0.1-9.1	3.8	81.1
(2014)	Mumbai (India)	σ _{ci} (MPa)	Uniaxial compression strength	41	2.2-181.7	84.5	66.4
Diamantis et al		I _{s50} (MPa)	Point load index	32	1.04-4.93	3.08	35.1
(2009)	Mount Othrys (Greece)	σ _{ci} (MPa)	Uniaxial compression strength	32	19.2-125.7	60.3	44.8
Aggistalis et al		I _{s50} (MPa))	Point load index	62	0.34-4.54	1.75	54.6
(1980)	Epirus (Greece)	σ _{ci} (MPa)	Uniaxial compression strength	62	6.3-107.5	42.5	52.6

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		E _i (GPa)	Young's Modulus	62	0.96-9.84	4.45	50.5
Abdou and Mahmoud (2013)	Al-Jouf university (KSA)	σ _{ci} (MPa)	Uniaxial compression strength	30	26.0-36.2	28.8	7.7
Koncagül and	Proothitt (USA)	$\mathbf{S}_{\mathbf{h}}$	Shore Scleroscope hardness	31	14.9-47.6	26.6	34.8
Santi (1999)	breatha (05/4)	σ _{ci} (MPa)	Uniaxial compression strength	31	30.7-99.5	65.8	22.3
Ceryan et al. (2012)	Trabzon (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	55	7.3-24.1	14.1	27.2
Begonha and Sequeira Braga	Oporto (Portugal)	σ _{ci} (MPa)	Uniaxial compression strength	48	60-157	98.7	23.2
(2002)		E _i (GPa)	Young's Modulus	48	5.03-16.94	9.99	27.7
		γ (kN/m ³)	Unit weight	21	19.5-23.9	21.9	6.1
Shalabi et al. (2007)	Chicago (USA)	R _L	Schmidt hammer hardness	21	24-45	33.0	21.9
(2007)		σ _{ci} (MPa)	Uniaxial compressive strength	21	21.4-96.6	53.6	42.2
	– Donetsk (Ukraine)	n (%)	Porosity	15	27.5-47.2	36.2	15.7
Palchik (1999)		σ _{ci} (MPa)	Uniaxial compressive strength	15	7.1-19.8	12.7	29.1
		E _i (GPa)	Young's Modulus	15	1.4-2.5	1.92	19.5
		n (%)	Porosity	27	5.6-10.1	7.5	18.9
		$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	27	49-98	76.1	17.1
Bell and Lindsay	Durhan (South Africa)	σ _{bt} (MPa)	Brazilian tensile strength	27	6-20	14.9	24.4
(1999)	Durban (South Africa)	I _{s50} (MPa)	Point load index	27	3-13	9.0	30.3
		σ _{ci} (MPa)	Uniaxial compressive strength	27	77-214	136.6	25.6
	-	E _i (GPa)	Young's Modulus	27	10.9-99.9	52.3	51.4
		n (%)	Porosity	25	10.2-20.5	13.5	18.4
Bell (1978)	England (UK)	$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	28	24-60	42.6	23.4
	-	σ_{bt} (MPa)	Brazilian tensile strength	27	2.1-9.5	6.6	26.9

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		I _{s50} (MPa)	Point load index	28	0.2-9.5	4.68	54.1
		σ _{ci} (MPa)	Uniaxial compressive strength	29	33.2-112.4	78.4	30.8
		E _i (GPa)	Young's Modulus	18	19.7-46.2	32.7	20.3
		γ (kN/m ³)	Unit weight	10	23.8-25.1	24.6	1.5
Bell et al. (1997)	England (UK)	σ _{ci} (MPa)	Uniaxial compressive strength	10	25.7-45.4	35.5	17.9
Ghosh and		I _{s50} (MPa)	Point load index	11	2.04-5.88	3.65	42.7
Srivastava (1991)	Chamba (India)	σ _{ci} (MPa)	Uniaxial compressive strength	11	25-119	58.2	50.6
		n (%)	Porosity	20	3.4-17.5	8.6	44.9
Heidari et al		I _{s50} (MPa)	Point load index	20	3.17-5.56	4.23	16.0
(2012)	Hamedan (Iran)	σ _{ci} (MPa)	Uniaxial compressive strength	20	44.9-92.5	69.8	23.2
		E _i (GPa)	Young's Modulus	18	6.1-12.4	8.4	23.4
		γ (kN/m ³)	Unit weight	10	25.9-26.2	25.9	0.4
	Texas (USA)	n (%)	Porosity	10	4-14	7.9	40.1
		V _P (km/s)	P-wave velocity	10	4.50-5.13	4.81	4.5
		γ (kN/m ³)	Unit weight	17	26-26.8	26.2	1.1
	Texas (USA)	n (%)	Porosity	17	0.87-9.5	4.3	49.4
		V _P (km/s)	P-wave velocity	17	4.51-5.54	5.11	5.3
Li-h. (1001)		γ (kN/m ³)	Unit weight	13	23.9-26.5	25.2	3.0
JIZDA (1991)	Texas (USA)	n (%)	Porosity	13	4-13.5	7.9	34.8
		V _P (km/s)	P-wave velocity	13	4.06-4.90	4.57	5.5
		γ (kN/m ³)	Unit weight	15	25.9-26.7	26.2	1.2
	Texas (USA)	n (%)	Porosity	15	4.9-20.0	9.8	41.7
		V _P (km/s)	P-wave velocity	15	4.05-5.00	4.46	6.3
	Toyog (USA)	γ (kN/m ³)	Unit weight	12	25.9-27.6	26.3	1.8
	iexas (USA)	n (%)	Porosity	12	1.2-6.6	4.3	37.3

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		V _P (km/s)	P-wave velocity	10	5.21-5.86	5.32	3.6
		γ (kN/m ³)	Unit weight	18	22.9-27.1	25.3	7.0
Arslan et al. (2015)	Salt Range (Pakistan)	R _L	Schmidt hammer hardness	18	19-44	33.7	27.4
()		σ _{ci} (MPa)	Uniaxial compressive strength	18	42-108	74.3	30.8
		σ_{bt} (MPa)	Brazilian tensile strength	20	2.5-8.7	5.4	34.8
Palchik and		I _{s50} (MPa)	Point load index	18	1.69-4.28	2.61	28.7
Hatzor (2004)	Adulam (Israel)	σ _{ci} (MPa)	Uniaxial compressive strength	12	20.9-63.3	47.6	28.6
		E _i (GPa)	Young's Modulus	12	9.3-20.5	14.8	25.3
		γ (kN/m ³)	Unit weight	27	25.7-27.2	26.	1.4
Moradian and	Ghareh Tikan (Iran)	V _P (km/s)	P-wave velocity	27	1.84-6.54	5.05	30.1
Behnia (2009)		σ _{ci} (MPa)	Uniaxial compressive strength	27	40.7-143.1	82.2	31.6
		E _i (GPa)	Young's Modulus	27	13.7-90.5	48.9	47.8
Sharma and Singh (2007)	Jharia (India)	σ _{ci} (MPa)	Uniaxial compressive strength	10	22-28	25.6	6.7
	_	γ (kN/m ³)	Unit weight	25	25.2-27.5	26.3	2.3
		n (%)	Porosity	25	2.4-10.4	5.3	37.2
Pappalardo	Taormina (Italy)	V _P (km/s)	P-wave velocity	25	3.3-6.32	5.17	16.9
(2014)		σ _{ci} (MPa)	Uniaxial compressive strength	25	15.2-112	75.0	39.2
		E _i (GPa)	Young's Modulus	25	2.4-18.8	11.2	50.0
		γ (kN/m ³)	Unit weight	20	26.5-27.2	26.8	0.7
		n (%)	Porosity	20	0.06-0.4	0.22	39.6
Mishra and Basu		V _P (km/s)	P-wave velocity	20	5.36-6.25	5.82	4.7
(2013)	Malanjkhand (India)	σ _{bt} (MPa)	Brazilian tensile strength	20	10.5-19.8	15.5	15.6
		I _{s50} (MPa)	Point load index	19	5.66-14.13	9.02	23.8
	_	σ _{ci} (MPa)	Uniaxial compressive strength	20	91.5-201.7	150.1	18.9

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	Jharkhand (India)	γ (kN/m ³)	Unit weight	20	26.9-28.6	27.7	1.8
		n (%)	Porosity	20	0.2-0.54	0.39	26.6
		V _P (km/s)	P-wave velocity	20	5.12-6.25	5.84	5.8
		σ _{bt} (MPa)	Brazilian tensile strength	20	6.14-19.5	12.3	29.8
		I _{s50} (MPa)	Point load index	20	1.15-7.42	3.79	40.9
		σ _{ci} (MPa)	Uniaxial compressive strength	20	21.4-95.1	46.5	40.8
		γ (kN/m ³)	Unit weight	20	21.3-25.5	23.0	5.4
		n (%)	Porosity	19	2.9-15.5	9.8	44.1
		V _P (km/s)	P-wave velocity	20	2.73-4.99	3.62	20.5
		σ _{bt} (MPa)	Brazilian tensile strength	20	2.0-14.3	6.4	55.9
		Is50 (MPa)	Point load index	18	1.25-11.49	5.03	59.8
		σ _{ci} (MPa)	Uniaxial compressive strength	20	12.8-172.0	57.4	72.4
Gorski et al. (2007)	Forsmark (Sweden)	V _P (km/s)	P-wave velocity	20	4.93-5.21	5.07	1.5
		σ _{bt} (MPa)	Brazilian tensile strength	20	17.2-22.1	19.4	7.1
Dinçer et al. (2008)	- Adana (Turkey) -	γ (kN/m ³)	Unit weight	18	17.2-22.9	19.6	8.1
		n (%)	Porosity	18	16.2-32.5	25.0	17.2
		R _L	Schmidt hammer hardness	18	16.9-40.3	26.3	24.3
		$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	18	8.4-24.6	13.4	35.3
		V _P (km/s)	P-wave velocity	18	0.44-1.58	0.81	44.7
		I _{s50} (MPa)	Point load index	18	0.78-2.08	1.21	33.7
		σ _{ci} (MPa)	Uniaxial compressive strength	18	2.7-10.4	5.6	40.7
		E _i (GPa)	Young's Modulus	18	0.18-1.40	0.62	58.4
Khaksar et al. (1999)	Cooper Basin (Australia)	γ (kN/m ³)	Unit weight	22	20.9-25.1	23.2	5.3
		n (%)	Porosity	22	2.6-16.6	9.7	44.8
Kahraman and	Attendorn (Germany)	σ _{ci} (MPa)	Uniaxial compressive	24	9.8-86.6	32.8	69.2

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Alber (2006)			strength					
	-	E _i (GPa)	Young's Modulus	24	3-16.8	9.6	38.3	
Dinçer et al. (2004)	- Bodrum Peninsula (Turkey)	γ (kN/m ³)	Unit weight	10	17.5-26.5	21.4	17.9	
		R _L	Schmidt hammer hardness	10	24.8-53.4	35.4	27.7	
		σ _{ci} (MPa)	Uniaxial compressive strength	10	32.9-108	59.8	45.9	
		E _i (GPa)	Young's Modulus	10	5.1-21.2	10.6	53.2	
Rajabzadeh et al. (2011)	Dehbid (Iran)	n (%)	Porosity	14	0.30-0.86	0.46	36.4	
	- Neyqiz (Iran)	n (%)	Porosity	24	0.17-2.89	0.52	113.6	
		σ _{bt} (MPa)	Brazilian tensile strength	10	4.4-10.6	6.7	32.2	
		σ _{ci} (MPa)	Uniaxial compressive strength	10	43.1-101.8	63.0	26.3	
		E _i (GPa)	Young's Modulus	10	5.9-15.9	11.6	30.0	
Koçkar and Akgün (2003a)	Antalya (Turkey)	σ _{ci} (MPa)	Uniaxial compressive strength	12	28-117	73.3	42.9	
Nicksiar and Martin (2012)	Oskarshamn (Sweden)	σ _{ci} (MPa)	Uniaxial compressive strength	10	171-294	226.9	13.8	
		E _i (GPa)	Young's Modulus	10	72-80	75.7	3.8	
Basu et al. (2008)	Sao Paulo (Brazil)	R _L	Schmidt hammer hardness	20	20.6-55.4	45.6	17.6	
		$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	20	28.3-65.3	54.9	15.5	
		V _P (km/s)	P-wave velocity	20	1.93-5.51	4.51	18.3	
		σ _{ci} (MPa)	Uniaxial compressive strength	20	73-214	151.9	28.5	
		E _i (GPa)	Young's Modulus	20	41.1-70.2	58.9	15.3	
Basu and Kamran (2010)	Jharkhand (India)	I _{s50} (MPa)	Point load index	15	1.08-5.93	3.58	40.0	
		σ _{ci} (MPa)	Uniaxial compressive strength	15	40.1-107.9	77.4	24.0	
Gupta and Sharma (2012)	Himalaya (India)	$\gamma (kN/m^3)$	Unit weight	18	17.5-26.5	21.4	17.9	
		n (%)	Porosity	18	0.3-1.5	0.66	40.3	
		$V_{\rm p}(\rm km/s)$	P-wave velocity	18	1 48-5 53	3 84	31.5	
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	-	v p (km 3)	Unievial compressive	10	1.40 5.55	5.04	51.5	
		σ_{ci} (MPa)	strength	18	46-141	88.6	31.1	
		V _P (km/s)	P-wave velocity	20	4.11-5.29	4.84	7.1	
Kurtulus et al	-	I _{s50} (MPa)	Point load index	20	2.41-7.85	5.50	27.5	
(2011)	Ezine (Turkey)	σ _{ci} (MPa)	Uniaxial compressive strength	20	32.7-114.3	81.5	29.7	
		E _i (GPa)	Young's Modulus	20	3.4-5.4	4.5	13.5	
		γ (kN/m ³)	Unit weight	14	23.3-29.1	26.8	5.6	
Samun et al		V _P (km/s)	P-wave velocity	14	3.26-4.71	3.83	12.7	
(2010)	Afyonkarahisar (Turkey)	σ _{ci} (MPa)	Uniaxial compressive strength	14	13.2-57.1	34.7	40.6	
		E _i (GPa)	Young's Modulus	14	18.4-47.1	30.9	24.3	
	Siwalik (India)	γ (kN/m ³)	Unit weight	10	20.0-23.3	21.2	6.0	
		V _P (km/s)	P-wave velocity	10	2.10-2.54	2.28	8.0	
		σ _{ci} (MPa)	Uniaxial compressive strength	10	20.1-48.6	32.4	30.9	
	Gondwana (India)	γ (kN/m ³)	Unit weight	10	22.5-23.6	22.9	1.5	
Sarkar et al.		V _P (km/s)	P-wave velocity	10	2.35-2.64	2.49	3.9	
(2011)		σ _{ci} (MPa)	Uniaxial compressive strength	10	39.0-45.5	41.2	5.7	
		γ (kN/m ³)	Unit weight	10	21.9-23.6	22.5	2.6	
	Deccan Trap (India)	V _P (km/s)	P-wave velocity	10	2.15-3.02	2.54	14.6	
		σ _{ci} (MPa)	Uniaxial compressive strength	10	50.3-73.2	60.2	15.9	
		γ (kN/m ³)	Unit weight	10	24.5-26.5	25.5	2.7	
Coulour (1	-	V _P (km/s)	P-wave velocity	10	2.55-3.85	3.15	13.4	
(2010)	Himachal Pradesh (India)	I _{s50} (MPa)	Point load index	10	2.81-3.91	3.35	11.0	
	-	σ _{ci} (MPa)	Uniaxial compressive strength	10	68.4-84.5	75.2	8.0	
Tahir et al. (2011)	Kohat (Pakistan)	σ _{bt} (MPa)	Brazilian tensile strength	15	3.99-6.82	4.91	13.7	

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	_	I _{s50} (MPa)	Point load index	15	1.50-2.89	1.93	16.3
_		σ _{ci} (MPa)	Uniaxial compressive strength	15	26.6-49.0	38.0	14.7
		σ_{bt} (MPa)	Brazilian tensile strength	15	5.54-7.89	6.97	9.8
	Cherat (Pakistan)	Is50 (MPa)	Point load index	15	1.95-2.70	2.26	11.6
	, , <u> </u>	σ _{ci} (MPa)	Uniaxial compressive strength	15	29.4-61.8	49.8	18.7
		n (%)	Porosity	20	1.3-18.5	7.9	73.2
Cervan et al		V _P (km/s)	P-wave velocity	20	2.06-5.32	3.61	30.6
(2008)	Kurtun (Turkey)	σ _{ci} (MPa)	Uniaxial compressive strength	20	2.8-200.3	74.9	89.5
		E _i (GPa)	Young's Modulus	15	4-37.5	17.1	59.6
		n (%)	Porosity	10	2.2-2.6	2.4	5.7
	-	V _P (km/s)	P-wave velocity	10	4.20-4.70	4.46	4.3
Kurtulus et al	-	σ _{bt} (MPa)	Brazilian tensile strength	10	5.0-6.2	5.6	6.6
(2016)	Akveren (Turkey) –	Is50 (MPa)	Point load index	10	2.88-3.32	3.12	5.1
		σ _{ci} (MPa)	Uniaxial compressive strength	10	28-33	30	6.5
		E _i (GPa)	Young's Modulus	10	49-65	54.4	8.7
	Nigeria	σ _{ci} (MPa)	Uniaxial compressive strength	10	59.8-99.8	82.8	18.5
Adebayo and Umeh (2007)	Shagamu (Nigeria)	σ _{ci} (MPa)	Uniaxial compressive strength	10	60.3-97.4	82.5	17.6
	Lagos (Nigeria)	σ _{ci} (MPa)	Uniaxial compressive strength	10	67.3-119.2	96.8	21.1
		n (%)	Porosity	18	16.2-17.2	16.8	1.5
Chitty et al. (1994)	Salem (USA)	σ _{ci} (MPa)	Uniaxial compressive strength	14	46-59.3	51.1	8.5
		E _i (GPa)	Young's Modulus	14	25.2-29.6	26.9	5.5
Pittino et al. (2016)	Tauern Window (Austria)	V _P (km/s)	P-wave velocity	13	2.34-3.53	2.94	16.2
Kumari et al. (2016)	Strathbogie (Australia)	σ _{ci} (MPa)	Uniaxial compressive strength	11	76.3-143.3	118.2	18.4

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		E _i (GPa)	Young's Modulus	11	6.3-9.1	8.0	12.4
Chen and Hsu (2001)	Hualien (Taiwan)	σ _{bt} (MPa)	Brazilian tensile strength	12	3.45-10.97	7.65	31.0
Ulusay et al.	Kozly Zonguldak (Turkay) -	I _{s50} (MPa)	Point load index	15	2.2-4.0	3.14	18.1
(1994)	Koziu-Zoliguidak (Turkey)	E _i (GPa)	Young's Modulus	15	17-112	59.1	48.8
Mustafa et al. (2015)	Azad Kashmir (Pakistan)	I _{s50} (MPa)	Point load index	13	0.52-4.54	2.32	44.7
		γ (kN/m ³)	Unit weight	11	24.5-26.6	25.8	2.3
Shalabi et al. (2007)	Puerto Pico (USA) $-$	S_h	Shore scleroscope hardness	11	49-71	57.5	11.9
	r dento Kico (USA)	σ _{ci} (MPa)	Uniaxial compressive strength	11	11.2-55.1	32.3	38.5
		E _i (GPa)	Young's Modulus	11	16.2-41.1	26.4	30.4
	Detroit (USA)	γ (kN/m ³)	Unit weight	14	20.4-25.9	23.5	6.5
		R _L	Schmidt hammer hardness	14	23-45	34.7	17.3
		σ _{ci} (MPa)	Uniaxial compressive strength	14	19.9-109.9	57.7	46.7
	– Bodrum Peninsula (Turkey) –	γ (kN/m ³)	Unit weight	14	18.3-25.2	22.5	10.2
Dinçer et al.		R _L	Schmidt hammer hardness	14	27.9-52.4	43.1	17.5
(2004)		σ _{ci} (MPa)	Uniaxial compressive strength	14	38.5-112.7	82.5	25.7
		E _i (GPa)	Young's Modulus	14	7.8-28.3	13.6	25.8
Koçkar and Akgün (2003a)	Antalya (Turkey)	σ _{ci} (MPa)	Uniaxial compressive strength	15	3-104	50.9	66.5
		I _{s50} (MPa)	Point load index	10	5.0-7.9	6.5	13.7
Marques et al.	Drogil	I _{s50} (MPa)	Point load index	10	5.1-10.7	8.0	21.3
(2014)	Brazii –	Is50 (MPa)	Point load index	10	1.8-5.1	3.6	23.7
	-	I _{s50} (MPa)	Point load index	11	3.2-9.0	4.6	34.5
Sarkar et al.	Llimashal Duadach (India)	γ (kN/m ³)	Unit weight	10	25.9-27.6	26.6	2.3
(2010)	Himachal Pradesh (India) –	V _P (km/s)	P-wave velocity	10	3.05-4.26	3.68	12.5

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		Is50 (MPa)	Point load index	10	1.2-2.3	1.7	22.6
	-	σ _{ci} (MPa)	Uniaxial compressive strength	10	24.2-49.3	37.4	22.9
		γ (kN/m ³)	Unit weight	10	25.8-26.5	26.1	1.0
		V _P (km/s)	P-wave velocity	10	3.52-4.23	3.85	6.2
		Is50 (MPa)	Point load index	10	3.8-5.3	4.6	11.4
	-	σ _{ci} (MPa)	Uniaxial compressive strength	10	93.2-112.3	99.4	6.8
		γ (kN/m ³)	Unit weight	10	25.7-26.4	26.0	1.0
		V _P (km/s)	P-wave velocity	10	2.14-2.49	2.27	5.2
		Is50 (MPa)	Point load index	10	1.0-1.8	1.3	20.7
	_	σ _{ci} (MPa)	Uniaxial compressive strength	10	30.3-28.5	24.1	11.6
		γ (kN/m ³)	Unit weight	24	20.2-34.7	27.0	15.8
	Nusajaya, Desa Tebrau and Mersing, Johor (Malaysia)	V _P (km/s)	P-wave velocity	24	1.25-3.91	2.13	26.5
Mohamad et al.		σ _{bt} (MPa)	Brazilian tensile strength	24	0.8-4.2	2.35	46.2
(2014)		Is50 (MPa)	Point load index	24	0.3-4.1	1.9	70.2
		σ _{ci} (MPa)	Uniaxial compressive strength	24	5.5-61.1	27.7	65.3
	_	R _L	Schmidt hammer hardness	19	52.7-61.3	57.8	5.2
Vasconcelos et al.	Portugal -	V _P (km/s)	P-wave velocity	19	1.90-4.78	3.08	28.8
(2007)		σ _{ci} (MPa)	Uniaxial compressive strength	19	26-159.8	92.0	47.0
		E _i (GPa)	Young's Modulus	19	11.0-63.8	34.4	56.0
Ramana and		γ (kN/m ³)	Unit weight	10	29.4-31.0	30.1	1.9
Venkatanarayana (1973)	Mysore State (India)	V _P (km/s)	P-wave velocity	10	5.22-6.65	6.03	7.8
		E _i (GPa)	Young's Modulus	10	59.1-109.3	85.9	19.5
Nazir et al.	Malaysia –	R _L	Schmidt hammer hardness	20	28.9-39	35.1	7.5
(2013a)	Malaysia –	σ _{ci} (MPa)	Uniaxial compressive strength	20	52.2-85.6	71.6	12.8

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		γ (kN/m ³)	Unit weight	12	16.3-29.2	24.3	17.2
Kabraman (2001)	Turkov	R _L	Schmidt hammer hardness	26	8.1-65.8	40.6	33.2
Kalifalliali (2001)	-	I _{s50} (MPa)	Point load index	28	0.23-12.01	2.99	91.4
		σ _{ci} (MPa)	Uniaxial compressive strength	28	4.4-152.7	50.5	66.8
Nazir et al.		σ _{bt} (MPa)	Brazilian tensile strength	20	3.02-14.2	7.16	36.1
(2013b)	Malaysia	σ _{ci} (MPa)	Uniaxial compressive strength	20	21.2-100.7	59.6	31.9
		R _L	Schmidt hammer hardness	12	23-50	38.3	22.1
$\mathbf{P}_{\text{compon}}$ (1000)	UV	σ _{bt} (MPa)	Brazilian tensile strength	12	3.84-18.42	11.80	40.3
Dearman (1999)	UK	σ _{ci} (MPa)	Uniaxial compressive strength	12	47.8-274.8	141.9	46.3
	-	E _i (GPa)	Young's Modulus	12	15.9-64.2	44.1	36.1
Zhao and Li	C:	σ _{bt} (MPa)	Brazilian tensile strength	23	7.7-17.1	11.6	21.5
(2000)	Singapore	E _i (GPa)	Young's Modulus	22	25.7-56.	40.2	19.9
	-	n (%)	Porosity	13	0.18-13.3	4.3	105.2
		R _L	Schmidt hammer hardness	13	32.1-56.	16.9	18.0
Kahraman et al.		V _P (km/s)	P-wave velocity	13	3.7-6.2	5.24	17.6
(2004)	Turkey	σ _{bt} (MPa)	Brazilian tensile strength	13	2.2-10.2	5.18	39.6
		σ _{ci} (MPa)	Uniaxial compressive strength	13	45.4-175.0	91.8	45.2
		I _{s50} (MPa)	Point load index	13	1.6-7.1	4.7	31.3
		γ (kN/m ³)	Unit weight	10	16.7-27.1	22.6	21.5
Balci et al. (2004)	Turkey	σ _{bt} (MPa)	Brazilian tensile strength	10	1.2-11.6	5.3	64.5
2 der et al. (2001)	<u> </u>	σ _{ci} (MPa)	Uniaxial compressive strength	10	10.8-173.6	68.2	80.2
~		γ (kN/m ³)	Unit weight	15	20.8-31.3	26.6	11.1
Singh et al. (2012)	India	V _P (km/s)	P-wave velocity	15	2.15-3.67	2.81	17.3
(2012)		σ _{ci} (MPa)	Uniaxial compressive	15	29-61.8	44.0	22.6

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			strength				
Kasim and Shakoor (1996)	US	σ _{ci} (MPa)	Uniaxial compressive strength	22	34-209.6	127.6	38.3
Klanphumeesri (2010)	Thailand	σ_{bt} (MPa)	Brazilian tensile strength	15	7.8-11.8	9.9	14.4
		n (%)	Porosity	22	0.06-10.7	2.68	107.3
		R _L	Schmidt hammer hardness	15	44.3-56.5	48.5	6.9
(2005)	Turkey	σ_{bt} (MPa)	Brazilian tensile strength	11	5.7-18.1	11.2	42.1
()		I _{s50} (MPa)	Point load index	22	2.9-13.3	8.1	47.7
		σ _{ci} (MPa)	Uniaxial compressive strength	22	26.1-210.6	108.9	50.2
		γ (kN/m ³)	Unit weight	25	19.0-26.6	24.2	9.24
Verwaal and		n (%)	Porosity	21	0.4-37.9	8.86	115.1
Mulder (1993)	Netherlands -	σ _{ci} (MPa)	Uniaxial compressive strength	25	22-203	106.7	56.5
		E _i (GPa)	Young's Modulus	25	9-80	45.0	49.6
	Sutto (Hungary)	γ (kN/m ³)	Unit weight	27	23.9-25.4	24.7	1.4
Török and Vásárhelyi (2010)		σ _{ci} (MPa)	Uniaxial compression strength	27	66.8-124.3	93.8	21.8
		σ _{ci} (MPa)	Uniaxial compression strength	32	19.2-125.7	60.3	44.8
Pittino et al. (2016)	Diyarbakir (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	28	24.0-90.5	43.6	37.9
		σ _{ci} (MPa)	Uniaxial compression strength	27	77.0-214.0	136.6	25.6
Bell and Lindsay (1999)	Durban (South Africa)	$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	27	49-98	76.1	17.1
		σ_{bt} (MPa)	Brazilian tensile strength	20	6-20	14.9	24.4
Bell (1978)	England (UK)	σ _{bt} (MPa)	Brazilian tensile strength	27	2.1-9.5	6.6	26.9
Bell et al. (1997)	England (UK)	σ _{ci} (MPa)	Uniaxial compression strength	10	25.7-45.4	35.5	17.9
		γ (kN/m ³)	Unit weight	10	23.8-25.1	24.6	1.5

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Ghosh and	Chamba (India)	σ _{ci} (MPa)	Uniaxial compression strength	10	25.0-83.3	52.1	43.5
Srivastava (1991)		I _{s50} (MPa)	Point load index	10	2.04-5.47	3.43	42.2
Heidari et al.	Hamedan (Iran)	σ _{ci} (MPa)	Uniaxial compression strength	20	44.9-82.5	69.8	23.2
(2012)		E _i (GPa)	Young's Modulus	20	3.17-5.56	4.23	16.0
		σ _{ci} (MPa)	Uniaxial compression strength	12	20.9-63.3	47.6	28.6
Palchik and	Adulam (Israel)	Is50 (MPa)	Point load index	18	1.69-4.28	2.61	28.7
Hatzor (2004)		E _i (GPa)	Young's Modulus	12	9.3-20.5	14.8	25.3
		σ _{bt} (MPa)	Brazilian tensile strength	20	2.5-8.7	5.4	34.8
Moradian and Behnia (2009)	Ghareh Tikan (Israel)	σ _{ci} (MPa)	Uniaxial compression strength	27	40.7-143.1	82.2	31.6
		γ (kN/m ³)	Unit weight	27	25.7-27.2	26.4	1.4
Pappalardo	Castelmola (Italy)	σ _{ci} (MPa)	Uniaxial compression strength	21	47.3-112.0	84.0	26.3
(2014)		γ (kN/m ³)	Unit weight	25	25.2-27.5	26.3	2.3
	Malanjkhand (India)	γ (kN/m ³)	Unit weight	20	16.49-27.17	16.81	0.7
		σ _{ci} (MPa)	Uniaxial compression strength	20	91.5-201.7	150.1	18.9
		Is50 (MPa)	Point load index	19	5.66-14.13	9.02	23.8
Mishra and Basu		σ _{bt} (MPa)	Brazilian tensile strength	20	10.5-19.8	15.5	15.6
(2013)		γ (kN/m ³)	Unit weight	20	26.88-28.55	27.70	1.8
		σ _{bt} (MPa)	Brazilian tensile strength	20	6.14-17.47	12.16	29.8
	Jharkhand (India)	I _{s50} (MPa)	Point load index	20	1.15-7.42	3.79	40.9
		σ _{ci} (MPa)	Uniaxial compression strength	20	21.36-95.14	46.53	40.8
Gorski et al. (2007)	Forsmark (Sweden)	σ _{bt} (MPa)	Brazilian tensile strength	20	17.2-22.1	19.4	7.1
Dincer et al		γ (kN/m ³)	Unit weight	18	17.32-22.94	19.63	8.1
Dinçer et al. (2008)	Adana (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	18	2.65-10.41	5.63	40.7

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		I _{s50} (MPa)	Point load index	18	0.78-2.08	1.21	33.7
		E _i (GPa)	Young's Modulus	18	0.18-1.4	0.62	58.4
		R _L	Schmidt hammer hardness	18	16.9-40.3	26.3	24.3
		$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	18	8.4-24.6	13.4	35.3
		γ (kN/m ³)	Unit weight	18	17.3-22.9	19.6	8.1
Kahraman and Alber (2006)	Attrndorn (Germany)	E _i (GPa)	Young's Modulus	22	4.6-15.8	9.52	33.4
Rajabzadeh et al.	Namia (Inan)	E _i (GPa)	Young's Modulus	10	5.9-15.9	11.6	28.0
(2011)	Neyriz (Iran)	σ _{bt} (MPa)	Brazilian tensile strength	10	4.4-10.6	6.7	32.2
Koçkar and Akgün (2003a)	Antalya (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	12	28-117	73.3	42.9
Nicksiar and	Oskarshamn (Sweden)	σ _{ci} (MPa)	Uniaxial compression strength	10	171-294	226.9	13.8
Martin (2012)	(1.1.1.1)	E _i (GPa)	Young's Modulus	10	72-80	75.7	3.8
$\mathbf{P}_{\text{act}} = 1 (2008)$	Sao Paulo (Brazil)	R _L	Schmidt hammer hardness	19	36.24-55.38	46.91	11.9
Dasu et al. (2008)		$\mathbf{S}_{\mathbf{h}}$	Shore scleroscope hardness	19	44.95-65.32	56.31	10.6
Basu and Kamran	Jharkhand (India)	σ _{ci} (MPa)	Uniaxial compression strength	15	40.1-107.9	77.4	24.0
(2010)		Is50 (MPa)	Point load index	15	1.08-5.93	3.58	40.0
Gupta and	Himalaya (India)	σ _{ci} (MPa)	Uniaxial compression strength	18	46.0-141.0	88.6	31.1
Sharma (2012)		γ (kN/m ³)	Unit weight	18	25.5-27.7	26.4	2.5
Kurtulus et al		σ _{ci} (MPa)	Uniaxial compression strength	20	32.7-114.3	81.5	29.7
(2011)	Ezine (Turkey)	Is50 (MPa)	Point load index	20	2.41-7.85	5.50	27.5
	-	E _i (GPa)	Young's Modulus	20	3.4-5.4	4.5	13.5
Sarpun et al.	Afvonkarahisar (Turkev)	σ _{ci} (MPa)	Uniaxial compression strength	14	13.2-57.1	34.7	40.6
(2010)	j · (E _i (GPa)	Young's Modulus	14	18.4-47.1	31.0	24.3

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		γ (kN/m ³)	Unit weight	14	23.2-29.1	26.8	5.6
Sarkar et al. (2010)	Himachal Pradesh (India)	I _{s50} (MPa)	Point load index	10	2.81-3.91	3.35	11.0
		σ _{ci} (MPa)	Uniaxial compression strength	15	26.6-49.0	38.0	14.8
	Kohat (Pakistan)	Is50 (MPa)	Point load index	15	1.5-2.9	1.93	16.3
Tahir et al. (2011)	_	σ _{bt} (MPa)	Brazilian tensile strength	15	4.0-6.8	4.9	13.7
_		σ _{ci} (MPa)	Uniaxial compression strength	15	29.4-61.8	49.8	18.7
	Cherat (Pakistan)	I _{s50} (MPa)	Point load index	15	2.0-2.7	2.26	11.6
		σ _{bt} (MPa)	Brazilian tensile strength	15	5.5-7.9	6.9	9.8
		σ _{ci} (MPa)	Uniaxial compression strength	10	28-33	30.0	6.5
Kurtulus et al.	Akveren (Turkey)	Is50 (MPa)	Point load index	10	2.9-3.3	3.12	5.1
(2016)	-	E _i (GPa)	Young's Modulus	10	49-65	54.4	8.7
		σ _{bt} (MPa)	Brazilian tensile strength	10	5.0-6.2	5.6	6.6
_	Nigeria	σ _{ci} (MPa)	Uniaxial compression strength	10	59.8-99.8	82.8	18.5
Adebayo and Umeh (2007)	Shagamu (Nigeria)	σ _{ci} (MPa)	Uniaxial compression strength	10	60.3-97.4	82.5	17.6
	Lagos (Nigeria)	σ _{ci} (MPa)	Uniaxial compression strength	10	67.3-119.2	96.8	21.1
Chitty et al. (1994)	Salem (USA)	σ _{ci} (MPa)	Uniaxial compression strength	14	46.0-59.3	51.1	8.5
Ulusay et al.	Kozlu-Zonguldak (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	15	55.0-96.0	70.7	15.8
(1994)		I _{s50} (MPa)	Point load index	15	2.2-4.0	3.14	18.1
Shelehi et el	_	σ _{ci} (MPa)	Uniaxial compression strength	11	11.2-55.1	32.3	38.5
(2007)	Puerto Rico (USA)	E _i (GPa)	Young's Modulus	11	16.2-41.1	26.4	30.4
		$\gamma (kN/m^3)$	Unit weight	11	24.5-26.6	25.8	2.3
Dinçer et al. (2004)	Bodrum Peninsula (Turkey)	σ _{ci} (MPa)	Uniaxial compression strength	12	70-112.7	89.2	15.2

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		E _i (GPa)	Young's Modulus	12	9.3-18.3	14.6	19.5
	-	γ (kN/m ³)	Unit weight	14	18.3-25.1	22.5	10.2
	Turkey	R _L	Schmidt hammer hardness	24	24.80-53.40	39.9	23.1
Dincer et al. (2004)		σ _{ci} (MPa)	Uniaxial compression strength	24	32.93-112.7	73.05	35.7
(2004)		E _i (GPa)	Youngs' modulus	24	5.05-21.18	12.34	37.7
_		γ (kN/m ³)	Unit weight	24	17.45-26-50	22.04	13.7
		n (%)	Porosity	204	0.2-29.3	4.66	104.3
		γ (kN/m ³)	Unit weight	468	14-26.3	23.22	12.7
	- Graaca (Sandstana)	R _L	Schmidt hammer hardness	280	11-52	28.9	37.4
	Greece (Sandstone) -	I _{s50} (MPa))	Point load index	828	0.2-7.6	1.88	91.5
		σ _{ci} (MPa)	Uniaxial compression strength	226	2.5-252	48.53	108.4
Sabatakakis et		E _i (GPa)	Youngs' modulus	36	3.42-71.75	26.36	66.31
al. (2008)	- 	n (%)	Porosity	262	0.04-4.25	0.54	105
		γ (kN/m ³)	Unit weight	778	22-28.2	26.17	2.9
		Ν	Schmidt hammer hardness	355	16-57	41.8	13.8
		I _{s50} (MPa)	Point load index	1305	2-7	3.99	28.3
	-	σ _{ci} (MPa)	Uniaxial compression strength	470	25-294.05	68.23	65.3
		E _i (GPa)	Youngs' modulus	85	4.7-196.26	73.17	73.7
		w _c (%)	Water content	15	0.01-0.04	0.02	50.00
	– Otanmäki, Finland (Gabbro)	n (%)	Porosity	15	0.13-0.48	0.29	34.48
	· · · · · · · · -	σ _{bt} (MPa)	Brazilian tensile strength	15	6.8-12.6	9.88	18.02
Aladejare (2020) –		w _c (%)	Water content	10	0.02-0.12	0.05	80.00
	- Otanmäki, Finland (Granite)	n (%)	Porosity	10	0.29-1.64	0.73	58.90
	-	σ _{bt} (MPa)	Brazilian tensile strength	10	7.1-12.7	10.41	18.83

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No. of Property Site Description Property Range of data Property COV (%) Source tests mean (≥30) Orange-Fish Tunnel (South ROD Rock quality designation 44 56.4-99.1 86.6 12.8 Africa) Dworshak Dam (USA) 21 Bieniawki (1978) ROD Rock quality designation 59.6-93.2 82.1 12.5 Orange-Fish Tunnel (South E_m(GPa) Modulus of deformation 7 0.22-1 0.69 41.4 Africa) Sapigni et al. Maen Tunnel (Italy) RMR 330 13-96 24.7 Rock mass rating 67.7 (2002)42 47.9 RQD Rock quality designation 10-100 69.7 The Divriği open-pit mine Özkan et al. (Turkey) RMR Rock mass rating 39 51-80 64.5 11.6 (2015)The Divriği open-pit mine RMR Rock mass rating 35 50-72 60.7 11.6 (Turkey) Nejati et al. Gotvand Dam (Iran) RMR Rock mass rating 49 30-76 50.5 23.0 (2013)Hassanpour et al. Karaj Water Conveyance Tunnel RMR Rock mass rating 37 36-74 56.0 13.9 (2009)(Iran) Tumbler Ridge Tunnels Kaiser et al. 20-71 RMR 26.8 Rock mass rating 49 49.0 (1986)(Canada) Rock mass rating RMR 43 14.5-70 39.6 38.0 Platamon Railway Tunnel Chatziangelou et Geological strength al. (2002) (Greece) GSI 43 37.7 24.5 22.5-52.5 index RQD Rock quality designation 80 58-94 80.2 11.1 Waikato Coal Region (New Zealand) RMR 51.2-72 64.8 Rock mass rating 65 6.6 Moon et al. (2001)32 RMR Jinzhou (China) Rock mass rating 56-78 71.9 7.8 RMR 74 61.7 Iran Rock mass rating 39-85 20.0 RQD Rock quality designation 61 15-100 77.9 29.8 Chun et al. Korea (2009)RMR 61 21-92 62.3 23.2 Rock mass rating Coon (1968) Rock quality designation Dworshak (USA) RQD 40 35-92 74.18 16.9 Majdi and Beiki Geological strength GSI 22.4 Iran 111 26-89 54.3 (2010)index

 Table 1.A4. Site-specific rock mass property statistics (database: ROCKMass/9/5876)

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	\mathbf{D}_{1} , \mathbf{D}_{2} (Cline)	RMR	Rock mass rating	23	67-92	81.2	8.6
Yanjun et al. (2007)	Daya Bay (China) —	Q	Rock mass quality	11	33.6-57.6	46.9	17.6
	Jinzhou (China)	RMR	Rock mass rating	32	56-78	71.9	7.8
Kramadibrata et al. (2011)	Tutupan open pit coal mine (Indonesia)	RMR	Rock mass rating	22	23-71	44.3	35.2
Khabbazi et al. (2013)	Iron	RQD	Rock quality designation	10	30-85	63.9	27.7
	IIdli	RMR	Rock mass rating	10	39-56	488.1	11.9
	Iran	RMR	Rock mass rating	74	39-85	61.7	20.0
$E1 N_{aga} (1006)$	Wadi Muiih (Jordan)	RMR	Rock mass rating	16	48-63	56	8.0
EI-Ivaqa (1996)	wadi Mujib (Jordan) —	Q	Rock mass quality	16	2.8-18.3	10.2	53.9
	The Bushkoppies Tunnel (South	RMR	Rock mass rating	24	38-87	70.1	18.7
	Africa)	Q	Rock mass quality	24	0.02-200	13.4	303.5
Cameron-Clarke	The Du Toitskloof Pilot Tunnel (South Africa)	RMR	Rock mass rating	22	30-81	54.2	28.5
(1981)		Q	Rock mass quality	22	0.09-89.7	10.1	209.2
	The Delvers Street Tunnel	RMR	Rock mass rating	10	28-74	45.9	32.4
	(South Africa)	Q	Rock mass quality	10	0.01-4.75	1.37	112.3
Tučenil (1008)	A totilal Dom (Tualcov)	RMR	Rock mass rating	21	13-42	29.0	31.1
Tugrui (1998)	Ataturk Dam (Turkey) —	Q	Rock mass quality	21	0.05-1.9	0.64	87.5
Kumar et al.	III:	RMR	Rock mass rating	29	34-63	46.1	17.1
(2017)	Himachai Pradesh (India) —	GSI	Geological strength index	29	17-33	23.1	15.4
		RMR	Rock mass rating	23	14-58	30.6	32.8
Hashemi et al. (2009)	Borujen (Iran)	GSI	Geological strength index	23	22-60	43.4	18.6
(2007)		Q	Rock mass quality	23	0.005-2.5	0.36	159.3
	Orange–Fish Tunnel (South Africa)	RQD	Rock quality designation	44	56.4-99.1	86.6	12.8
Bieniawski (1978)	Dworshak Dam (USA)	RQD	Rock quality designation	21	59.6-93.2	82.1	12.5
(17/0)	Orange-Fish Tunnel (South Africa)	E _m (GPa)	Modulus of deformation	7	0.22-1	0.69	41.4
El-Naqa and Al	Tannur Dam (Jordan)	RQD	Rock quality designation	11	44-57	49.1	9.1

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Kuisi (2002)		RMR	Rock mass rating	11	52-60	56.1	4.8
	-	GSI	Geological strength index	11	48-57	52	5.4
	-	Q	Rock mass quality	11	2.43-5.92	3.98	29.5
-		RQD	Rock quality designation	17	50.3-100	89.9	15.0
	Taiz (Yemen)	RMR	Rock mass rating	17	52.1-80.3	67.8	12.1
	-	GSI	Geological strength index	17	33.6-72	58.2	20.6
Aialloeian and		RMR	Rock mass rating	28	41-70	57.8	14.9
Mohammadi	– Khersan II Dam (Iran)	GSI	Geological strength index	28	37-64	48.0	15.4
(2013)	-	E _m (GPa)	Modulus of deformation	28	3.4-40.5	21.3	58.6
Al-Quadhi and		RQD	Rock quality designation	12	85.2-100	95.8	4.8
Janardhana	Taiz (Yemen)	RMR	Rock mass rating	12	62.8-80.3	72.1	6.7
(2016)	-	GSI	Geological strength index	12	52.9-72	64.5	8.8
Jordá-Bordehore et al. (2016)	Mirador (Ecuador)	RQD	Rock quality designation	15	41-100	84.1	18.0
Jordá-Bordehore	Sucre, Isabela (Ecuador)	Q	Rock mass quality	12	4.3-12.2	7.4	36.9
et al. (2016): Jordá-Bo	Chato (Ecuador)	RQD	Rock quality designation	10	10-86	75.4	30.7
rdehore (2017)		Q	Rock mass quality	10	0.3-15	4.93	114.7
	Deimisian (Ermedan)	RQD	Rock quality designation	15	35-100	78.3	30.6
Jordá-Bordehore	Primicias (Ecuador) –	Q	Rock mass quality	15	0.22-100	22.9	133.3
(2017)	Cueve de Les Vendes (Spein)	RQD	Rock quality designation	27	76-100	87.9	7.1
	Cueva de Los verdes (Spain) –	Q	Rock mass quality	27	2.8-150	29.9	138.5
		RMR	Rock mass rating	31	31-80	55.7	19.0
Kumar (2002)	Himalaya	RQD	Rock quality designation	15	55-75	65.7	9.8
	_	GSI	Geological strength index	12	32-67	53.6	18.2
		RQD	Rock quality designation	21	46.9-78.1	64.7	11.8
Zolfaghari et al. (2015)	Bakhtiary dam site (Iran)	Q	Rock mass quality	21	0.92-5.02	2.44	57.3
(2013)	-	E _m (GPa)	Modulus of deformation	21	2.9-5.5	4.4	25.6

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Kayur et al		RMR	Rock mass rating	19	32-77	53.2	22.8
(2015)	Karun3 (Iran) —	E _m (GPa)	Modulus of deformation	19	1.2-54	12.5	101.7
	Shrestha (2005) Khimti tunnel (Nepal) —		Rock quality designation	26	10-50	25.6	51.5
Shrestha (2005)			Rock mass quality	26	0.005-0.60	0.13	118.4
Hassanpour et al. (2009)	Karaj Water Conveyance Tunnel (Iran)	RMR	Rock mass rating	37	36-74	56.0	13.9
Kitagawa et al. (1991)	Nou Tunnel (Japan)	RQD	Rock quality designation	26	11.2-80.3	36.9	54.3
Exadaktylos et	West Dail Line (Hong Kong)	RMR	Rock mass rating	12	62.5-78.5	69.6	6.4
al. (2008)	west Kan Line (Hong Kong) —	Q	Rock mass quality	12	2.5-79.2	22.8	120.2
Bagde et al. (2002)	Dongargaon fluorite mine (India)	RMR	Rock mass rating	14	23-52	37.3	20.3
del Potro and Hürlimann (2008)	Tenerife (Spain)	GSI	Geological strength index	26	39-80	53.9	20.3
Khanlari et al. (2012)	Karaj–Tehran tunnel (Iran)	RMR	Rock mass rating	24	21-75	20.3	29.0
Pavlovic (1970)	Unknown	E _m (GPa)	Modulus of deformation	27	3.5-45.1	30.6	39.0
	Dave Dave (Ching)	RMR	Rock mass rating	23	67-92	81.2	8.6
Yanjun et al. (2007)	Daya Bay (China) —	Q	Rock mass quality	23	33.6-180.5	74.3	60.0
(2007)	Jinzhou (China)	RMR	Rock mass rating	32	56-78	71.9	7.8
Kramadibrata et al. (2011)	Tutupan open pit coal mine (Indonesia)	RMR	Rock mass rating	22	23-71	44.3	35.2
		RQD	Rock quality designation	26	34.9-100	85.1	19.0
	Chungcheong-do and Kyungsang-do (Korea)	RMR	Rock mass rating	26	46-86	71.5	16.3
		E _m (GPa)	Modulus of deformation	26	3.02-35.7	17.9	51.8
Chun et al. (2006)	Chungcheong-do and	RQD	Rock quality designation	18	13.1-99.8	71.4	38.4
(2000)	Kyungsang-do (Korea)	RMR	Rock mass rating	18	43-94	66.7	20.8
	Chungcheong-do and	RQD	Rock quality designation	23	35.9-99.9	75.5	26.6
	Kyungsang-do (Korea)	RMR	Rock mass rating	22	32-84	65	24.1
Koçkar and	Ilıksu tunnels (Turkey)	RMR	Rock mass rating	27	31-59	45.1	29.4

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Akgün (2003b)		Q	Rock mass quality	27	0.13-7.31	3.03	98.5
		RMR	Rock mass rating	17	18-54	40.8	29.0
Kaffalar (2014)		GSI	Geological strength index	14	10-50	34.3	44.3
Kelleler (2014)	Nevada (USA) —	Q	Rock mass quality	17	0.008-6.6	1.12	158.2
		E _m (GPa)	Modulus of deformation	14	0.001-1.03	0.29	103.0
	The Huai Saphan Hin Power	RQD	Rock quality designation	10	32-99	81	25.8
Ъ.,	(HSHP) Tunnel (Thailand)	Q	Rock mass quality	10	0.04-6.6	3.16	92.1
Ranasooriya (2009)	The Ramboda Pass Highway Tunnel (Sri Lanka)	RMR	Rock mass rating	19	30-73	56.2	24.6
	The Namroud Water Resources Project Diversion Tunnel (Iran)	RMR	Rock mass rating	11	17-40	26.9	34.0
Birid (2014)	Mumbai (India)	RMR	Rock mass rating	22	9-67	35.0	46.8
Alexander (1960)	Turmut (Australia)	E _m (GPa)	Modulus of deformation	18	4.98-12.0	8.1	23.9
Singh (2011)		RQD	Rock quality designation	24	73.2-87.5	81.9	7.8
	Baspa Hydroelectric project (India)	Q	Rock mass quality	24	1.7-7.6	5.2	49.7
		E _m (GPa)	Modulus of deformation	24	3.6-13.9	7.2	42.7
		RQD	Rock quality designation	10	9.8-100	55.7	59.4
Isik et al. (2008)	Ankara (Turkey)	GSI	Geological strength index	12	8-53	22.2	57.0
		E _m (GPa)	Modulus of deformation	23	0.02-0.27	0.11	55.8
Tumac et al. (2006)	Küçüksu tunnel (Turkey)	RQD	Rock quality designation	15	75-90	87	6.1
Borsetto et al.	Timme grande Douronhouse (Italy)	RQD	Rock quality designation	189	0-100	54.6	54.4
(1983)	Thipagrande Fowerhouse (Italy) —	E _m (GPa)	Modulus of deformation	21	0.3-26.2	6.7	70.1
Frough et al. (2014)	Karaj-Tehran water conveyance tunnel (Iran)	RMR	Rock mass rating	10	21-75	53.2	31.4
Moradi and Farsangi (2013)	Zagros long water conveyance tunnel (Iran)	RQD	Rock quality designation	15	12.5-79	45	46.9
Panthi and Shrestha (2018)	Kaligandaki headrace tunnel (Nepal)	Q	Rock mass quality	14	0.02-2	0.75	106.1
Mayer and Stead (2016)	Ok Tedi mine site (Papua New Guinea)	GSI	Geological strength index	10	29-53	44.2	17.9

Swolfs and	South Table Mountain (USA)	RQD	Rock quality designation	10	16.4-98.6	70.8	50.1
Kibler (1982)	South Table Mountain (USA) -	E _m (GPa)	Modulus of deformation	10	12.9-43.2	27.8	39.8
Danielsen and	Hellendese Tunnel (Sweden)	RQD	Rock quality designation	12	12.5-62.5	37.5	51.3
Dahlin (2009)	Hallandsas Tulliel (Sweden)	Q	Rock mass quality	12	0.01-5.5	1.29	153.4
		RQD	Rock quality designation	16	15-95	57.6	47.9
Frough and Torabi (2013)	Karaj–Tehran tunnel (Iran)	RMR	Rock mass rating	24	21-75	50.3	29.0
101401 (2012)	_	Q	Rock mass quality	24	0.003-50	7.11	153.6
Jhanwar et al.	nwar et al.		Rock quality designation	11	40-65	49.1	24.0
(2000)	Doligh-Buzurg mine (mula) –	RMR	Rock mass rating	11	24-65	38.4	35.6
Lama and Vutukuri (1978)	Japanese Hydro Projects (Japan)	E _m (GPa)	Modulus of deformation	21	1.0-10.1	3.73	73.2
Jafari et al. (2007)	Nosoud water tunnel (Iran)	GSI	Geological strength index	11	21-56	37.2	33.3
Pinto da Cunha (1991)	Karun dam site (Iran)	E _m (GPa)	Modulus of deformation	11	6-57	26.9	73.7
Judeel (2003)	Klerksdorp gold field (South Africa)	GSI	Geological strength index	11	27-65	48.4	30.1
Radhakrishnan and Leung (1989)	Singapore	RQD	Rock quality designation	16	15-53.8	30.3	35.6
Taheri and Tani	Australia	GSI	Geological strength index	10	46-50	48.4	3.0
(2009)	Australia	GSI	Geological strength index	10	40-45	42.4	5.4

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2. Site-specific correlations between geotechnical properties

Yelu Zhou, Dongming Zhang, and Jianye Ching

2.1 Introduction

It is well known that soil parameters are generally "correlated" to each other. The existence of a large number of transformation models demonstrate the usefulness of bivariate relationships between soil parameters. The concept of a correlation expands the deterministic notion of a relationship, such as a mean trend, to a probabilistic notion that describes the strength of the relationship on top of the mean trend. In this report, the site-specific bivariate correlation coefficients are computed based on the multivariate soil databases shown in Table 2.1. Note that for the soil databases (clay and sand), most parameters do not have a unit (dimensionless), but for the rock and rock mass databases, most parameters have units. Most databases are generic (global), except that SH-CLAY/11/4051 is a municipal clay database of Shanghai. Three types of correlation coefficients are considered: (a) the Pearson product-moment correlation coefficient (τ); (b) the Spearman rank correlation coefficient (τ); and (c) the Kendall's tau rank correlation coefficient (τ).

Database	Reference	Parameters of interest	# data points	# sites/ studies
CLAY/10/7490	Ching and Phoon (2014)	LL, PI, LI, σ'_{v}/P_{a} , σ'_{p}/P_{a} , s_{u}/σ'_{v} , S_{t} , q_{t1} , q_{tu} , B_{q}	7490	251 studies
SAND/7/2794	Ching et al. (2017)	$D_{50}, C_u, D_r, \sigma'_v/P_a, \phi', Q_{tn}, (N_1)_{60}$	2794	176 studies
ROCK/9/4069	Ching et al. (2018)	$\gamma,n,R_L,S_h,\sigma_{bt},I_{s50},V_p,\sigma_{ci},E_i$	4069	184 studies
ROCKMass/9/5876	Ching et al. (2020)	RQD, RMR, Q, GSI, Em, Eem, Edm, Ei, σ_{ci}	5784	225 studies
CLAY/8/12225	Ching (2020)	LL, PI, w, e, σ'_v/P_a , C _c , C _s , c _v	12225	427 studies
CLAY/12/3997	Ching (2020)	LL, PI, LI, σ_v/P_a , σ_p/P_a , s_u/σ_v , K_0 , E_u/σ_v , B_q , q_{t1} , $N_{60}/(\sigma_v/P_a)$	3997	237 studies
SAND/13/4113	Ching (2020)	e, Dr, σ'_{v}/P_{a} , σ'_{p}/P_{a} , K ₀ , E _{dn} , Q _{tn} , B _q , (N ₁) ₆₀ , K _{DMT} , E _{DMTn} , E _{PMTn} , M _n	4113	172 studies
SH-CLAY/11/4051	Zhang et al. (2020)	LL, PI, LI, e, K ₀ , σ'_{v}/P_{a} , su(UCST)/ σ'_{v} , su(VST)/ σ'_{v} , St(UCST), St(VST), p_{v}/σ'_{v}	4051	50 sites in Shanghai

Table 2.1. Soil/rock databases

 γ = unit weight; ϕ' = effective friction angle; σ_p = preconsolidation stress; σ_v = vertical effective stress; σ_{bt} = Brazilian tensile strength; σ_{ci} = uniaxial compressive strength of intact rock; $(N_1)_{60} = N_{60}/(\sigma'_v/P_a)^{0.5}$; $B_q = CPT$ pore pressure ratio = $(u_2-u_0)/(q_t-\sigma_v)$; C_c = compression index; C_s = swelling index; C_u = coefficient of uniformity; c_v = coefficient of consolidation; D_{50} = median grain size; D_r = relative density; e = void ratio; E_{DMT} = soil modulus determined by DMT; E_{DMTn} = normalized E_{DMT} = $(E_{DMT}/P_a)/(\sigma'_v/P_a)^{0.5}$; E_{PMT} = soil modulus determined by PMT; E_d = drained modulus of sand; E_{PMTn} = normalized E_{PMT} = (E_{PMT}/P_a)/(σ_v/P_a)^{0.5}; E_{dn} = $(E_d/P_a)/(c_v/P_a)^{0.5}$; E_{dm} = dynamic modulus of rock mass; E_{em} = elasticity modulus of rock mass; E_i = Young's modulus of intact rock; E_m = deformation modulus of rock mass; E_u = undrained modulus of clay; GSI = geological strength index; I_{s50} = point load strength index for diameter 50 mm; K_0 = at-rest lateral earth pressure coefficient; K_{DMT} = dilatometer horizontal stress index; LI = liquidity index; LL = liquid limit; n = porosity; M = effective constrained modulus determined by oedometer; M_n = normalized M = $(M/P_a)/(\sigma_v/P_a)^{0.5}$; N₆₀ = corrected SPT-N; P_a = atmospheric pressure = 101.3 kPa; PI = plasticity index; p_s = specific penetration resistance from the CPT (unique to China); Q = Q-system; $q_c = cone$ tip resistance; $q_t = corrected$ cone tip resistance; $Q_{tn} =$ $(q_t/P_a)/(\sigma_v/P_a)^{0.5}$; $q_{t1} = (q_t - \sigma_v)/\sigma_v =$ normalized cone tip resistance; $q_{tu} = (q_t - u_2)/\sigma_v =$ effective cone tip resistance; $R_L = L$ -type Schmidt hammer hardness; RMR = rock mass rating; RQD = rock quality designation; S_h = Shore scleroscope hardness; SPT-N = standard penetration test blow count; S_t = sensitivity; s_u = undrained shear strength for clay; s_u^{re} = remoulded s_u ; u_0 = hydrostatic pore pressure; $u_2 = CPTU$ pore pressure; UCST = unconfined compression soil test; $V_p = P$ -wave velocity; VST = vane shear test; w = water content.

2.2 Summary Tables

Tables 2.2-2.6 summarize some site-specific correlation coefficients between different parameter pairs for the databases in Table 2.1. All correlation coefficients in this report are site-specific in the

State-of-the-art review of inherent variability and uncertainty, March 2021 sense that they are computed for each site, not for the entire soil/rock database.

2.3 Key observations

- 1. Figure 2.1 compares the site-specific ρ coefficients for the global vs. Shanghai municipal databases. It shows that the site-specific ρ 's for the Shanghai database mostly span in a narrower range than those for the global database.
- 2. Evans (1996) classified ρ into 5 categories based on its magnitude. The correlation is very strong when $|\rho| \ge 0.8$, strong when $0.6 \le |\rho| < 0.8$, moderate when $0.4 \le |\rho| < 0.6$, weak when $0.2 \le |\rho| < 0.4$, and very weak when $|\rho| < 0.2$. The results in Tables 2.1-2.5 are shaded by different colors according to the ρ median value (ρ_{median}): red means very strong, orange means strong, yellow means moderate, green means weak, and blue means very weak.
- 3. Figure 2.2 compares the histograms of site-specific ρ_{median} for normalized vs. non-normalized parameter pairs in ROCK/9/4069 (e.g., I_{s50} -E_i vs. I_{s50}/σ_{ci} -E_i/ σ_{ci}). It shows that |site-specific ρ_{median} | decreases after normalization.
- 4. Figure 2.3 compares the histograms of site-specific ρ_{median} for global vs. Shanghai municipal sites. It shows that the site-specific ρ 's at a municipal scale seem stronger than those at a global scale.
- 5. Figure 2.4 compares the histograms of site-specific ρ_{median} between non-dimensional parameters for clay vs. rock. There is no clear difference between soil and rock in the general trends.



a)



Figure 2.1. Boxplots of site-specific ρ for: a) LL-PI; b) LL- K_0 ; c) LL- s_u/σ'_v ; d) LI- S_t ; e) LI- K_0 ; f) LI- s_u/σ'_v ; g) ω - K_0 ; h) ω - s_u/σ'_v . The lower and upper edges of the box mean first and third quartiles (25% at 75% percentiles), whereas the lower and upper bars mean the further extensions of the above mentioned quartiles by 1.5 times of IQR (IQR = $\rho_{75\% \text{ percentile}} - \rho_{25\% \text{ percentile}}$)



Figure 2.2. The histograms of site-specific ρ_{median} for normalized vs. non-normalized parameter pairs in ROCK/9/4069



Figure 2.3. The histograms of site-specific ρ_{median} for global vs. Shanghai municipal clay sites



Figure 2.4. The histograms of site-specific ρ_{median} of normalized parameters for clay vs. rock

2.4 References

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		No	of				Value		
Property	No.	tests/	/site	Type of			value		
nairs	of	10515/	5110	correlation	2 504	2504		750/	07.5%
pans	sites	Range	Mean	conclation	2.3%	23%	Median	7.5%	97.5%
	70	10 (20	20	TZ 1 11	percentile	percentile	0.50	percentile	
~	/0	10-639	30	Kendali	-0.06	0.29	0.52	0.66	0.91
w-C _c	70	10-639	30	Pearson	-0.37	0.43	0.72	0.86	0.98
	70	10-639	30	Spearman	-0.07	0.38	0.70	0.83	0.98
	18	10-35	16	Kendall	-0.40	0.02	0.34	0.48	0.73
$w-C_s$	18	10-35	16	Pearson	-0.58	0.16	0.33	0.68	0.90
	18	10-35	16	Spearman	-0.58	0.08	0.37	0.66	0.89
	99	10-57	16	Kendall	-0.70	-0.24	-0.05	0.24	0.72
w-su/ σ'_v	101	10-57	16	Pearson	-0.92	-0.38	-0.06	0.35	0.85
	99	10-57	16	Spearman	-0.83	-0.35	-0.05	0.32	0.84
	21	10-41	18	Kendall	-0.59	-0.08	0.20	0.48	1.00
$w-K_0$	23	10-41	17	Pearson	-0.91	-0.13	0.01	0.54	1.00
	21	10-41	18	Spearman	-0.72	-0.14	0.29	0.64	1.00
	41	10-50	15	Kendall	-0.58	-0.29	-0.02	0.34	0.72
w-E ₂ /σ' ₂	41	10-50	15	Pearson	-0.94	-0.41	-0.07	0.58	0.82
	41	10-50	15	Spearman	-0.75	-0.33	-0.03	0.46	0.85
	263	10-623	20	Kendall	0.15	0.55	0.05	0.40	0.05
II_PI	203	10-623	20	Pearson	0.31	0.89	0.02	0.09	1.00
LL-1 1	213	10-025	20	Speerman	0.41	0.89	0.90	0.99	1.00
	<u> </u>	10-025	20	Vandall	0.01	0.84	0.94	0.97	0.99
	07	10-005	52 20	Rendan	-0.01	0.31	0.55	0.08	0.80
LL-C _c	67	10-605	32	Pearson	-0.07	0.47	0.67	0.87	0.97
	6/	10-605	32	Spearman	-0.04	0.42	0./1	0.86	0.96
	14	10-33	14	Kendall	-0.44	0.04	0.27	0.42	0.87
LL-C _s	14	10-33	14	Pearson	-0.51	0.01	0.46	0.58	0.95
	14	10-33	14	Spearman	-0.55	0.02	0.36	0.59	0.95
	86	10-57	16	Kendall	-0.70	-0.27	-0.02	0.25	0.84
$LL-s_u/\sigma'_v$	86	10-57	16	Pearson	-0.77	-0.40	0.00	0.31	0.85
	86	10-57	16	Spearman	-0.83	-0.35	-0.04	0.36	0.93
	20	10-41	19	Kendall	-0.32	-0.02	0.18	0.33	0.73
LL-K ₀	23	10-40	19	Pearson	-0.57	-0.02	0.12	0.39	0.88
	20	10-41	19	Spearman	-0.44	-0.04	0.21	0.49	0.83
	35	10-29	15	Kendall	-0.52	-0.19	0.02	0.44	0.72
$LL-E_{\rm u}/\sigma'_{\rm v}$	35	10-29	15	Pearson	-0.60	-0.40	0.06	0.50	0.84
u - v	35	10-29	15	Spearman	-0.69	-0.27	0.09	0.58	0.84
	65	10-605	27	Kendall	-0.02	0.26	0.42	0.63	0.87
PI-C.	65	10-605	27	Pearson	0.08	0.29	0.56	0.81	0.97
1100	65	10-605	27	Spearman	-0.01	0.35	0.50	0.77	0.96
	15	10-33	14	Kendall	-0.43	0.05	0.29	0.52	0.50
PL-C	15	10-33	14	Pearson	-0.46	0.12	0.25	0.52	0.07
1 I-C s	15	10-33	14	Spearman	-0.40	0.03	0.43	0.00	0.90
	15	10-55	14	Vandall	-0.32	0.03	0.41	0.08	0.80
IIC	43	10-115	19	Desman	-0.40	0.00	0.29	0.34	0.80
LI-S _t	45 45	10-113	19	Pearson	-0.57	0.12	0.55	0.68	0.87
	45	10-113	19	Spearman	-0.52	0.06	0.42	0.69	0.94
LLOCD	74	10-57	16	Kendall	-0.73	-0.25	-0.11	0.34	0.54
LI-OCR	11	10-57	16	Pearson	-0.79	-0.38	-0.03	0.29	0.83
	74	10-57	16	Spearman	-0.85	-0.39	-0.16	0.41	0.70
	64	10-57	16	Kendall	-0.69	-0.16	0.06	0.29	0.59
$LI-s_u/\sigma'_v$	64	10-57	16	Pearson	-0.88	-0.22	-0.04	0.33	0.78
	64	10-57	16	Spearman	-0.82	-0.26	0.10	0.39	0.78
LI-Ko	9	11-41	19	Kendall	-0 74	-0.33	-0.10	0.16	0 79

Table 2.2. Summary of site-specific correlations for CLAY/10/7490, CLAY/8/12225, and CLAY/12/3997

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	No. No. of tests/site			f						
Property	of	tests	/site	Type of						
pairs	sites	Range	Mean	correlation	2.5%	25%	Median	75%	97.5%	
		1141180			percentile	percentile	1.1001411	percentile	percentile	
	9	11-41	19	Pearson	-0.85	-0.41	-0.14	0.14	0.69	
	9	11-41	19	Spearman	-0.90	-0.43	-0.17	0.17	0.87	
	34	10-27	15	Kendall	-0.74	-0.39	-0.14	0.10	0.72	
$LI-E_u/\sigma'_v$	39	10-27	16	Pearson	-0.88	-0.49	-0.01	0.10	0.88	
	34	10-27	15	Spearman	-0.87	-0.6	-0.15	0.16	0.89	
	38	10-41	15	Kendall	-0.88	-0.72	-0.52	-0.24	0.44	
B _q -OCR	38	10-41	15	Pearson	-0.98	-0.90	-0.68	-0.38	0.50	
	38	10-41	15	Spearman	-0.95	-0.85	-0.70	-0.25	0.56	
	30	10-40	15	Kendall	-0.86	-0.50	-0.16	0.06	0.39	
B_q - s_u/σ'_v	30	10-40	15	Pearson	-0.93	-0.73	-0.03	0.10	0.67	
	30	10-40	15	Spearman	-0.96	-0.63	-0.12	0.06	0.58	
	25	10-24	16	Kendall	-0.76	-0.58	-0.28	-0.09	0.64	
B_q - E_u/σ'_v	25	10-24	16	Pearson	-0.88	-0.75	-0.55	-0.26	0.65	
	25	10-24	16	Spearman	-0.90	-0.76	-0.38	-0.13	0.78	
	44	10-60	16	Kendall	-0.40	0.24	0.48	0.74	0.92	
q _{t1} -OCR	44	10-60	16	Pearson	-0.47	0.44	0.82	0.93	0.99	
	44	10-60	16	Spearman	-0.56	0.30	0.62	0.89	0.98	
	90	10-59	15	Kendall	-0.76	-0.30	0.17	0.63	0.96	
$OCR-s_u/\sigma'_v$	92	10-59	15	Pearson	-0.89	-0.35	0.30	0.96	0.99	
	90	10-59	15	Spearman	-0.89	-0.38	0.25	0.81	0.99	
	37	10-40	17	Kendall	-0.44	0.41	0.79	0.86	0.97	
OCR-K ₀	39	10-40	17	Pearson	-0.49	0.42	0.81	0.93	0.98	
	37	10-40	17	Spearman	-0.53	0.48	0.87	0.96	0.99	
	25	10-50	15	Kendall	0.07	0.27	0.42	0.59	0.93	
OCR- $E_{\rm u}/\sigma'_{\rm v}$	25	10-50	15	Pearson	0.27	0.46	0.77	0.91	0.99	
	25	10-50	15	Spearman	0.10	0.38	0.59	0.74	0.98	
	37	10-59	16	Kendall	-0.59	-0.01	0.29	0.62	0.82	
$a_{t1}-s_{u}/\sigma'_{v}$	37	10-59	16	Pearson	-0.79	-0.13	0.59	0.91	0.99	
In a v	37	10-59	16	Spearman	-0.71	-0.05	0.40	0.79	0.94	
	7	13-43	19	Kendall	0.18	0.38	0.51	0.66	0.87	
a_{t1} -K ₀	8	12-43	18	Pearson	0.00	0.29	0.599	0.81	0.86	
-1110	7	13-43	19	Spearman	0.22	0.47	0.67	0.82	0.97	
	42	10-33	16	Kendall	-0.33	0.19	0.43	0.61	0.83	
$a_{t1}-E_{u}/\sigma'_{u}$	42	10-33	16	Pearson	-0.37	0.54	0.71	0.91	0.99	
	42	10-33	16	Spearman	-0.41	0.27	0.56	0.77	0.95	
	2.7	10-38	16	Kendall	-0.11	0.15	0.53	0.71	0.95	
s./σ'F./σ'	2.7	10-38	16	Pearson	-0.22	0.52	0.82	0.95	0.99	
	2.7	10-38	16	Spearman	-0.20	0.17	0.74	0.71	0.99	
	18	10-39	17	Kendall	-0.79	-0.46	-0.01	0.38	0.84	
s/σ'Ko	21	10-39	16	Pearson	-0.83	-0.44	0.00	0.53	0.98	
50/07110	18	10-39	17	Spearman	-0.87	-0.64	0.00	0.53	0.94	
	10	18-18	18	Kendall	0.05	0.04	0.05	0.05	0.05	
s/c/ C	1	18-18	18	Pearson	0.03	0.05	0.05	0.03	0.05	
Su/O v-Cc	1	18-18	18	Spearman	0.07	0.07	0.07	0.07	0.07	
	1	10.17	14	Kendall	-0.07	0.00	0.33	0.57	0.00	
E/or K	5	10-17	13	Pearson	_0.19	0.05	0.35	0.37	0.60	
$\mathbf{E}_{\mathbf{u}} \mathbf{O}_{\mathbf{v}} \mathbf{K}_{0}$	4	10-17	13	Spearman	_0.00	0.05	0.55	0.45	0.85	
	4	10-17	14	Kendall	_0.09	_0.10	_0.40	_0.71	_0.05	
K. C	1	10	10	Dearson	-0.21	-0.21	-0.21	-0.21	-0.21	
K0-Cc	1	10	10	Spearman	-0.28	-0.28	-0.28	-0.28	-0.28	
	22	10 115	10	Kondall	-0.28	-0.28	-0.28	-0.28	-0.28	
C _c -C _s	33 22	10-115	22	Deerser	-0.13	0.15	0.33	0.38	0.80	
	55	10-115	22	rearson	-0.48	0.20	0.40	0.75	0.90	

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Property	No.	No. tests/	of /site	Type of	Value					
pairs	of sites	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile	
	33	10-115	22	Spearman	-0.25	0.19	0.47	0.74	0.95	
Note: red m	eans ve	ry strong	, <mark>orang</mark>	ge means str	ong, yellov	w means n	noderate,	green mear	is weak and	
blue means	verv we	eak								

blue means very weak.

 Table 2.3. Summary of site-specific correlations for SH-CLAY/11/4051

Property	No.	No. of tests/site		Type of	Value					
pairs	of sites	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile	
	18	40-496	124	Kendall	0.78	0.82	0.84	0.86	0.96	
LL-PI	18	40-496	124	Pearson	0.94	0.95	0.97	0.98	1.00	
	18	40-496	124	Spearman	0.92	0.95	0.96	0.97	1.00	
	11	10-59	19	Kendall	0.29	0.48	0.56	0.71	0.86	
LL-K ₀	11	10-59	19	Pearson	0.40	0.61	0.80	0.88	0.97	
	11	10-59	19	Spearman	0.35	0.64	0.76	0.87	0.94	
/ .	14	10-38	24	Kendall	-0.67	-0.45	-0.22	-0.07	0.08	
$LL-s_u/\sigma'_v$	14	10-38	24	Pearson	-0.75	-0.63	-0.47	-0.34	-0.05	
	14	10-38	24	Spearman	-0.83	-0.64	-0.34	-0.18	0.07	
	11	10-59	19	Kendall	0.23	0.45	0.56	0.75	0.82	
PI-K ₀	11	10-59	19	Pearson	0.50	0.64	0.79	0.87	0.95	
	11	10-59	19	Spearman	0.32	0.63	0.75	0.88	0.93	
II St	12	10-38	26	Kendall	-0.17	-0.03	0.16	0.32	0.57	
LI-St	12	10-38	26 26	Pearson	-0.21	0.07	0.20	0.38	0.79	
	12	10-50	10	Vandall	-0.24	0.00	0.22	0.40	0.70	
I I-Ko	11	10-39	19	Rendan	-0.24	-0.05	0.15	0.54	0.48	
	11	10-59	19	Spearman	-0.18	-0.14	0.29	0.57	0.60	
	14	10-38	24	Kendall	-0.29	0.15	0.21	0.35	0.49	
$LI-s_u/\sigma'_v$	14	10-38	24	Pearson	-0.30	0.13	0.31	0.33	0.47	
	14	10-38	24	Spearman	-0.48	0.20	0.39	0.42	0.66	
	11	10-59	19	Kendall	0.33	0.49	0.60	0.73	0.90	
w-K ₀	11	10-59	19	Pearson	0.42	0.64	0.75	0.90	0.95	
	11	10-59	19	Spearman	0.41	0.64	0.78	0.88	0.97	
	18	10-41	23	Kendall	-0.45	-0.35	-0.10	0.05	0.33	
$w\text{-}s_u/\sigma'_v$	18	10-41	23	Pearson	-0.73	-0.38	-0.19	-0.03	0.37	
	18	10-41	23	Spearman	-0.66	-0.50	-0.15	0.06	0.41	
	1	12	12	Kendall	0.03	0.03	0.03	0.03	0.03	
K_0 -su/ σ'_v	1	12	12	Pearson	-0.05	-0.05	-0.05	-0.05	-0.05	
	1	12	12	Spearman	0.05	0.05	0.05	0.05	0.05	
	3	10-21	14	Kendall	-0.51	-0.42	-0.16	0.16	0.27	
p_s/σ'_v-K_0	3	10-21	14	Pearson	-0.80	-0.74	-0.56	0.04	0.25	
	3	10-21	14	Spearman	-0.70	-0.61	-0.35	0.19	0.37	
$p_s/\sigma'_v - s_u/\sigma'_v$	13	11-41	24	Kendall	-0.18	0.25	0.42	0.46	0.58	
	13	11-41	24	Pearson	0.06	0.61	0.68	0.85	0.94	

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Property	No.	No. of tests/site		Type of	Value						
pairs	of	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile		
	13	11-41	24	Spearman	-0.20	0.33	0.57	0.63	0.76		
Note: red m	ieans ve	ry stron	<mark>g</mark> , <mark>oran</mark> g	ge means sti	rong, yellov	w means n	noderate,	green meai	ns weak and		
h 1		a a 1-									

blue means very weak.

Table 2.4. Summary of site-specific correlations for SAND/7/2794 and SAND/10/4113

$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Property	No.	No.	of	Tomos	Value					
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	pairs	of sites	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		21	10-228	36	Kendall	0.53	0.62	0.73	0.83	0.91	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	D _r -Q _{tn}	21	10-228	36	Pearson	0.72	0.83	0.91	0.94	0.96	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		21	10-228	36	Spearman	0.69	0.81	0.88	0.94	0.98	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		4	12-72	40	Kendall	0.13	0.18	0.28	0.37	0.42	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	D _r -M _n	4	12-72	40	Pearson	-0.04	0.09	0.26	0.36	0.43	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		4	12-72	40	Spearman	0.16	0.26	0.40	0.50	0.56	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		5	12-72	38	Kendall	0.33	0.35	0.47	0.65	0.92	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	K ₀ -M _n	5	12-72	38	Pearson	0.71	0.76	0.85	0.92	0.98	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		5	12-72	38	Spearman	0.62	0.63	0.71	0.78	0.98	
$ \begin{array}{c cccccccccccccccccccccccccccccc$	(N ₁) ₆₀ -Q _{tn}	15	10-21	14	Kendall	-0.54	0.00	0.38	0.69	0.75	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		15	10-21	14	Pearson	-0.70	0.23	0.48	0.85	0.91	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		15	10-21	14	Spearman	-0.73	0.05	0.55	0.86	0.91	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		21	10-21	15	Kendall	-0.60	-0.03	0.20	0.46	0.82	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	(N ₁) ₆₀ -K _{DMT}	21	10-21	15	Pearson	-0.70	-0.08	0.34	0.64	0.90	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		21	10-21	15	Spearman	-0.76	-0.04	0.23	0.61	0.94	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		45	10-31	17	Kendall	-0.41	0.14	0.37	0.53	0.77	
45 10-31 17 Spearman -0.52 0.21 0.52 0.69 0.91 2 12-30 21 Kendall 0.43 0.43 0.52 0.61 0.61	Q_{tn} - K_{DMT}	45	10-31	17	Pearson	-0.57	0.16	0.55	0.75	0.89	
2 12-30 21 Kendall 0.43 0.43 0.52 0.61 0.61		45	10-31	17	Spearman	-0.52	0.21	0.52	0.69	0.91	
		2	12-30	21	Kendall	0.43	0.43	0.52	0.61	0.61	
$E_{DMTn}-M_n$ 2 12-30 21 Pearson 0.55 0.55 0.59 0.63 0.63	E _{DMTn} -M _n	2	12-30	21	Pearson	0.55	0.55	0.59	0.63	0.63	
2 12-30 21 Spearman 0.59 0.59 0.68 0.78 0.78		2	12-30	21	Spearman	0.59	0.59	0.68	0.78	0.78	
2 12-30 21 Kendall 0.54 0.54 0.59 0.64 0.64	K _{DMT} -M _n	2	12-30	21	Kendall	0.54	0.54	0.59	0.64	0.64	
K _{DMT} -M _n 2 12-30 21 Pearson 0.65 0.65 0.69 0.72 0.72		2	12-30	21	Pearson	0.65	0.65	0.69	0.72	0.72	
2 12-30 21 Spearman 0.71 0.71 0.76 0.81 0.81		2	12-30	21	Spearman	0.71	0.71	0.76	0.81	0.81	

Note: red means very strong, orange means strong, yellow means moderate, green means weak and

blue means very weak.

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Property	No.	No. tests	. of /site	Type of			Value		
pairs	sites	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile
	31	10-66	19	Kendall	-0.11	0.36	0.66	0.79	1.00
γ_{d} - σ_{ci}	31	10-66	19	Pearson	-0.26	0.51	0.83	0.93	0.99
	31	10-66	19	Spearman	-0.14	0.45	0.83	0.93	1.00
	11	10-66	24	Kendall	-0.04	0.17	0.29	0.61	0.75
γ_d - E_i	11	10-66	24	Pearson	-0.04	0.11	0.42	0.84	0.90
	11	10-66	24	Spearman	-0.09	0.27	0.41	0.78	0.90
γ_d -V _P	29	10-66	20	Kendall	-0.46	0.10	0.67	0.92	1.00
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 Table 2.5. Summary of site-specific correlations for ROCK/9/4069

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Property	No.	NO. tests/	or /site	Type of			Value		
pairs	of			correlation	2.5%	25%		75%	97.5%
1	sites	Range	Mean		percentile	percentile	Median	percentile	percentile
	29	10-66	20	Pearson	-0.55	0.14	0.83	0.93	0.99
	29	10-66	20	Spearman	-0.58	0.15	0.84	0.97	1.00
	11	10-45	20	Kendall	-0.65	-0.62	-0.52	-0.35	0.33
n-o _{bt}	11	10-45	20	Pearson	-0.90	-0.79	-0.70	-0.44	0.38
ii ou	11	10-45	20	Spearman	-0.84	-0.78	-0.68	-0.45	0.53
	26	10-55	25	Kendall	-0.84	-0.67	-0.54	-0.31	0.13
n C .	26	10-55	25	Pearson	-0.92	-0.88	-0.73	-0.45	0.05
II-0 _{C1}	20	10-55	25	Spearman	-0.92	-0.85	0.73	-0.45	0.19
	20	10-35	23	Kondoll	-0.90	-0.85	-0.75	-0.45	0.13
πE	17	10-45	24	Deemaan	-0.71	-0.03	-0.55	-0.30	0.02
II-Ei	17	10-45	24	Pearson	-0.91	-0.81	-0.72	-0.47	-0.11
	1/	10-45	24	Spearman	-0.90	-0.80	-0.72	-0.46	-0.01
X7	20	10-55	24	Kendall	-0.87	-0.76	-0.55	-0.34	0.16
n-v _P	20	10-55	24	Pearson	-0.95	-0.88	-0.72	-0.46	0.14
	20	10-55	24	Spearman	-0.96	-0.89	-0./1	-0.43	0.27
рт	9	13-145	41	Kendall	0.05	0.28	0.54	0.67	0.76
\mathbf{K}_{L} - \mathbf{I}_{s50}	9	13-145	41	Pearson	0.05	0.38	0.70	0.88	0.89
	9	10 145	41	Vandall	0.00	0.40	0.74	0.85	0.90
D.	19	10-145	30	Rendan	-0.13	0.42	0.59	0.75	0.82
K_L - σ_{ci}	19	10-145	30	Pearson	-0.21	0.60	0.82	0.90	0.97
	19	10-145	30	Spearman	-0.22	0.60	0.75	0.90	0.93
	9	10-44	25	Kendall	-0.21	0.39	0.57	0.74	0.74
R _L -E _i	9	10-44	25	Pearson	-0.27	0.65	0.78	0.89	0.93
	9	10-44	25	Spearman	-0.30	0.54	0.70	0.89	0.90
	8	13-145	40	Kendall	-0.33	0.35	0.57	0.71	0.80
$R_L - V_P$	8	13-145	40	Pearson	-0.44	0.55	0.76	0.89	0.97
	8	13-145	40	Spearman	-0.53	0.50	0.73	0.86	0.93
	20	10-45	22	Kendall	0.16	0.51	0.65	0.75	0.91
σ_{bt} - σ_{ci}	20	10-45	22	Pearson	0.19	0.70	0.82	0.92	0.96
	20	10-45	22	Spearman	0.21	0.67	0.76	0.87	0.96
	13	10-43	21	Kendall	-0.50	0.08	0.58	0.74	0.87
σ_{bt} -I _{s50}	13	10-43	21	Pearson	-0.51	0.05	0.81	0.90	0.94
	13	10-43	21	Spearman	-0.64	0.07	0.74	0.89	0.96
	8	10-36	20	Kendall	-0.28	0.32	0.51	0.72	0.81
σ_{bt} -VP	8	10-36	20	Pearson	-0.44	0.50	0.66	0.82	0.92
	8	10-36	20	Spearman	-0.40	0.48	0.68	0.87	0.94
	36	10-145	28	Kendall	-0.25	0.50	0.64	0.78	1.00
I_{s50} - σ_{ci}	36	10-145	28	Pearson	-0.46	0.68	0.83	0.92	0.99
	36	10-145	28	Spearman	-0.34	0.68	0.80	0.82	1.00
	12	10-62	30	Kendall	-0.05	0.25	0.43	0.63	0.79
I_{s50} - E_i	12	10-62	30	Pearson	-0.12	0.38	0.63	0.81	0.98
	12	10-62	30	Spearman	-0.04	0.34	0.59	0.79	0.93
T T	18	10-145	28	Kendall	-0.15	0.50	0.61	0.82	1.00
I_{s50} -V _P	18	10-145	28	Pearson	-0.20	0.63	0.78	0.95	0.99
	18	10-145	28	Spearman	-0.23	0.68	0.77	0.95	1.00
	35	10-66	26	Kendall	0.15	0.46	0.62	0.73	0.88
σ_{ci} -E _i	35	10-66	26	Pearson	-0.13	0.57	0.83	0.92	0.98
	35	10-66	26	Spearman	0.16	0.63	0.79	0.88	0.96
	35	10-145	28	Kendall	0.03	0.58	0.68	0.84	1.00
σ_{ci} -V _P	35	10-145	28	Pearson	0.05	0.73	0.89	0.94	0.99
	35	10-145	28	Spearman	0.03	0.76	0.86	0.94	1.00
$n-E_i/\sigma_{ci}$	17	10-45	24	Kendall	-0.48	-0.13	-0.05	0.18	0.54

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Property pairs	No.	No. of tests/site		Type of	Value						
	sites	Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile		
	17	10-45	24	Pearson	-0.77	-0.19	0.01	0.36	0.77		
	17	10-45	24	Spearman	-0.63	-0.14	-0.06	0.26	0.71		
R_L - E_i/σ_{ci}	9	10-44	25	Kendall	-0.35	-0.26	-0.14	0.09	0.34		
	9	10-44	25	Pearson	-0.63	-0.43	-0.20	0.09	0.53		
	9	10-44	25	Spearman	-0.45	-0.37	-0.23	0.05	0.41		
$R_L\text{-}I_{s50}/\sigma_{ci}$	9	13-145	41	Kendall	-0.15	-0.05	0.06	0.10	0.17		
	9	13-145	41	Pearson	-0.24	-0.07	0.13	0.28	0.31		
	9	13-145	41	Spearman	-0.23	-0.08	0.10	0.11	0.24		
$I_{s50}/\sigma_{ci}\text{-}E_i/\sigma_{ci}$	12	10-62	30	Kendall	-0.57	-0.02	0.10	0.28	0.45		
	12	10-62	30	Pearson	-0.80	0.04	0.21	0.42	0.60		
	12	10-62	30	Spearman	-0.79	0.04	0.13	0.40	0.63		
$\sigma_{bt}/\sigma_{ci}\text{-}I_{s50}/\sigma_{ci}$	13	10-43	22	Kendall	-0.21	0.05	0.24	0.32	0.85		
	13	10-43	22	Pearson	-0.15	0.22	0.46	0.83	0.94		
	13	10-43	22	Spearman	-0.36	0.07	0.36	0.45	0.93		
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Note: red means very strong, orange means strong, yellow means moderate, green means weak and

blue means very weak.

Table 2.6. Summary of site-specific correlations for ROCKMASS/9/5876

Property pairs	No. of sites	No. of tests/site		Type of	Value						
		Range	Mean	correlation	2.5% percentile	25% percentile	Median	75% percentile	97.5% percentile		
	13	10-146	37	Kendall	0.18	0.39	0.60	0.65	0.70		
RQD-RMR	13	10-146 37		Pearson	0.01	0.56	0.80	0.84	0.87		
	13	10-146	37	Spearman	0.23	0.52	0.75	0.79	0.85		
RQD-Q	15	10-70	22	Kendall	0.23	0.48	0.61	0.73	0.98		
	16	10-70	10-70 22 Pe		-0.80	0.41	0.65	0.75	0.85		
	15	10-70	22	Spearman	0.25	0.62	0.73	0.86	0.99		
RQD-E _m /E _i	9	10-146	34	Kendall	-0.83	0.26	0.39	0.55	0.57		
	9	10-146	34	Pearson	-0.92	0.37	0.50	0.68	0.69		
	9	10-146	34	Spearman	-0.94	0.36	0.55	0.70	0.72		
RMR-Q	20	10-330	43	Kendall	-0.01	0.43	0.53	0.72	0.89		
	21	10-330	41	Pearson	-0.06	0.29	0.56	0.76	0.99		
	20	10-330	43	Spearman	-0.05	0.56	0.70	0.88	0.96		
RMR-GSI	9	11-60	26	Kendall	0.05	0.58	0.69	0.82	0.87		
	9	11-60	26	Pearson	0.12	0.63	0.91	0.95	0.95		
	9	11-60	26	Spearman	0.05	0.63	0.81	0.93	0.96		
RMR-E _m	16	10-715	103	Kendall	0.09	0.46	0.60	0.72	0.76		
	16	10-715	103	Pearson	0.00	0.58	0.68	0.86	0.86		
	16	10-715	103	Spearman	0.11	0.62	0.75	0.89	0.91		
RMR-E _{em}	9	11-418	63	Kendall	0.09	0.23	0.50	0.62	0.76		
	9	11-418	63	Pearson	0.00	0.41	0.50	0.68	0.86		
	9	11-418	63	Spearman	0.11	0.36	0.66	0.78	0.91		
E_m - E_{em}	15	10-418	43.8	Kendall	0.63	0.78	0.85	0.89	0.93		
	16	10-418	42.75	Pearson	0.55	0.92	0.94	0.98	0.99		
	15	10-418	43.8	Spearman	0.79	0.90	0.96	0.97	0.99		

Note: red means very strong, orange means strong, yellow means moderate, green means weak and

blue means very weak.

3. Summary of random field model parameters of geotechnical properties

Armin W. Stuedlein, Brigid Cami, Diego Di Curzio, Sina Javankhoshdel, Shin-ichi Nishumura, Wojciech Pula, Giovanna Vessia, Yu Wang, and Jianye Ching

3.1 Introduction

Random field theory (RFT) represents a robust framework for the evaluation of spatial variability. Whereas regression analyses invoke classical statistics with the necessary and fundamental assumption that all data (e.g., in-situ measurements) exhibit identical likelihoods of representation and lack of correlation, RFT acknowledges and leverages the location-specific dependence of soil properties. Specifically, soil properties within any one depositional unit are autocorrelated. A given soil measurement of interest, g(z), may be separated into two components including a trend function, t(z), and a randomly fluctuating component, w(z), as:

$$g(z) = t(z) + w(z)$$
(3.1)

where z = depth and w(z) represents inherent soil variability. It should be noted that the assessment of random fields commonly requires conditioning to obtain weak stationarity (i.e., statistical homogeneity), which may be achieved through progressive detrending, differencing, or variance transformation of the measured soil parameter. Acceptable stationarity may be characterized by a conditioned dataset with constant mean and an autocovariance that is a singular function of separation distance (Fenton 1999; Cafaro and Cherubini 2002).

The inherent spatial variability of a measured soil property can be sufficiently characterized by its mean, the variance or coefficient of inherent variability (COV), and the scale of fluctuation (i.e., measure of autocorrelation length; VanMarcke 1977, 1983). The coefficient of inherent variability is defined as:

$$COV_{w}(z) = \frac{\sigma_{w}(z)}{t(z)}$$
(3.2)

where σ_w = standard deviation of the fluctuating component of a measured, stationary soil parameter. The scale of fluctuation, δ , or the lag distance within which soils exhibit relatively strong correlation is the third parameter required to completely describe a finite random field within the RFT framework. Smaller magnitudes of δ describe short autocorrelation lengths and more variable soil conditions, whereas larger δ indicate long-lived autocorrelation and more homogeneous soil conditions. The determination of COV_w and δ should proceed carefully in order to prevent over-fitting during conditioning of the data to achieve weak stationarity.

3.2 Summary Tables and Figures

Table 3.1 and Figures 3.1 and 3.2 summarize the typical ranges in the vertical and horizontal scale

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of fluctuation, δ , reported in the literature. Table 3.2 provides a comprehensive summary of previously reported δ . Table 3.3 provides a comprehensive summary of previously-reported vertical and horizontal COV_w. Table 3.1 and Figure 3.1 are extracted from Cami et al. (2020), whereas many cases in Table 3.2 are also extracted from Cami et al. (2020).

3.3 Key Observations

- 1. In general, far more data is available in the vertical direction owing to the difficulty in obtaining sufficient data in the horizontal direction to compute random field model parameters (Table 3.1, Figure 3.1).
- 2. Data for the vertical direction is generally considered more reliable than that in the horizontal direction, owing to the use of more sophisticated methods available for determining the autocorrelation length when data is plentiful.
- 3. The available data suggest that the soil autocorrelation length anisotropy (i.e., δ_h / δ_v) ranges from 3 to 500 (Figure 3.2), whereas Table 3.3 indicates no significant difference between the horizontal and vertical COV_w. The most typical value for δ_h / δ_v is around 10-20 (Figure 3.2).
- 4. Random field model parameters reported in the literature span a wide range in geotechnical properties; however, those associated with the cone penetration test (CPT) are most frequent owing to the ease with which data may be obtained with the CPT and frequency of data samples associated with any given sounding.

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Coll town		δ	_h (m)		$\delta_{\rm v}$ (m)				
Son type	# studies	Min	Max	Average	# studies	Min	Max	Average	
Alluvial	9	1.1	49	14.8	11	0.07	2.53	0.66	
Ankara clay	-	-	-	-	1	1	6.2	3.63	
Chicago clay	-	-	-	-	2	0.4	1.25	0.72	
Clay	17	0.14	92.4	24.43	24	0.06	12.7	2.47	
Clay, sand, silt mix	13	1	1546	152.38	28	0.07	21	1.65	
Hangzhou clay	-	-	-	-	1	0.5	0.77	0.65	
Marine clay	6	2	60	31.3	7	0.11	6	1.85	
Marine sand	1	55	55	55	4	0.08	7.2	1.77	
Offshore soil	5	14	67	34.71	9	0.05	9.1	2.37	
Over consolidated clay	-	-	-	-	2	0.6	2.55	1.38	
Sand	8	1.7	75	11.29	12	0.1	4	1.14	
Sensitive clay	2	30	46	38	3	2	4	3	
Silt	3	12.7	45.5	33.22	5	0.14	7.19	2.08	
Silty clay	6	5	45.4	30.26	13	0.095	6.47	1.58	
Soft clay	4	22.1	80	41.1	11	0.14	6	1.76	
Undrained engineered soil	-	_	_	-	23	0.3	2.7	1.43	
Water content	9	2	60	18.5	5	0.2	3	1.22	

Table 3.1. Ranges of δ_h and δ_v for various soil types (Cami et al. 2020)



Figure 3.1. Histograms for δ_h and δ_v (extracted from Cami et al. 2020)



Figure 3.2. Variation of vertical scale of fluctuation, δ_v , with horizontal scale of fluctuation, δ_h , for various in-situ tests and soil conditions

3.4 References

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Site Location	Geomorphology	Soil type	Type of Measurement	Parameter	# soundings	Data interval (m)	$SOF_{h}(m)$	SOF _z (m)	Method	Model	Reference
Taiwan	Alluvial plane	Loose sandy soils, cohesive soils, medium dense to dense sands and clay	СРТ	qc	71	0.05		0.1-3.9	ММ	SExp	Liu and Chen (2010)
		layer	СРТ	$\mathbf{f}_{\mathbf{s}}$	71	0.05		0.2-1.9			
Beaufort Region, Canada	artificial island	Filled sand in artificial island	СРТ	q _c	18	0.02	1.69-13.69	0.42-0.44	MM	SExp	Lloret-Cabot et al. (2014)
Taiyuan, China		Silty clay	SPT	N value	6		39.4				
Taiyuan, China		Clay	SPT	N value	6		45.1		MM		Li and Xie (2000)
Hangzhou, China		Silty clay	SPT	N value	7		34.45				
Tianjin, China		Muck clay	СРТ	q_c	25			0.18-0.54	MM		Guo et al. (2017)
Xian,		Loss	СРТ	q _c				0.22-1.13	MM		Zhang and Liu (2011)
China		LOCSS	СРТ	$\mathbf{f}_{\mathbf{s}}$				0.26-1.12	IVIIVI		Zhang and Elu (2011)
Tioniin		Muck clay	СРТ	q_{c}	65		6.53-14.83	0.158-1.0	мм		
China		Clay	СРТ	q _c	65		8.37	0.132-0.322	101101		Yan et al. (2009)
		Silty clay	СРТ	q _c	65		9.65	0.095-0.426	MM		
Ningbo, China		Mucky clay	СРТ	q_{c}	9			0.32-0.49	MM		Wang and Chen (2019)
Nanjing,		Silty clay	СРТ	q_c	57			0.25-0.39	мм		Xue (2011)
China		Silty sand	СРТ	q_{c}	57			0.25-0.58	IVIIVI		Xue (2011)
Hangzhou,		Silty clay	СРТ	$\mathbf{f}_{\mathbf{s}}$	12			0.25-0.83	мм		Wu et al. (2005)
China		Clay	СРТ	f_s	12			0.16-1.26			
Vinijang		Silty Clay	SPT	N value	109			0.41-0.82	_		
China		Silt	SPT	N value	109			0.46-0.75	MM		Luo et al. (2008)
		Sand	SPT	N value	109			0.56-0.94	ļ		
College Station, TX,		Clay	СРТ	q_{c}	6	0.02		0.1 - 0.55	MM	SExp	Kulatilake & Um (2003)

 Table 3.2. Summary of scales of fluctuation reported in the literature

USA											
	Sand site: Silty Sand	CPT	q _c	22	0.02		0.72 - 3.08	MM	SExp		
		Sand site: Clean Sand	CPT	qc	22	0.02		0.96 - 2.82	MM	SExp	
		Sand site: Clavey Sand	CPT	qc	22	0.02		1.56 - 3.72	MM	SExp	
		Sand site: Hard Clay	CPT	q _c	22	0.02		0.61 – 1.95	MM	SExp	
		Clay site: very stiff clay	CPT	qc	24	0.02		0.97 – 3.14	MM	SExp	
		Clay site: sand	СРТ	q _c	24	0.02		0.38 - 0.77	MM	SExp	
		Clay site: very stiff clay	CPT	qc	24	0.02		0.59 - 2.98	MM	SExp	
College		Clay site: hard clay	CPT	q _c	24	0.02		0.26 – 1.33	MM	SExp	Akkaya & Vanmarcke
Station, TX, USA	Sand site: Silty Sand	CPT	$\mathbf{f}_{\mathbf{s}}$	22	0.02		1.21 - 3.04	MM	SExp	(2003)	
		Sand site: Clean Sand	CPT	f_s	22	0.02		0.83 - 2.34	MM	SExp	
		Sand site: Clavey Sand	CPT	f _s	22	0.02		1.25 - 3.53	MM	SExp	
		Sand site: Hard Clay	CPT	f_s	22	0.02		0.36 - 1.93	MM	SExp	-
		Clay site: very stiff clay	CPT	f_s	24	0.02		3.34 - 12.00	MM	SExp	
		Clay site: sand	CPT	$\mathbf{f}_{\mathbf{s}}$	24	0.02		0.91 - 2.93	MM	SExp	
		Clay site: very stiff clay	СРТ	\mathbf{f}_{s}	24	0.02		0.34 - 0.78	MM	SExp	1
		Clay site: hard clay	CPT	f_s	24	0.02		0.30 - 1.55	MM	SExp	
		Clay	CPT	q _c	11	0.049		0.85 - 6.13	SVR	Sph	
Missouri, USA		Clay	CPT	q _t	11	0.049		0.91 - 4.94	SVR	Sph	
		Clay	Sample	Cc		0.305		0.88 - 3.05	SVR	Sph	Onyejekwe & Ge
		Clay	Sample	e ₀		0.305		0.55 - 4.66	SVR	Sph	(2013)
		Clay	Sample	γ		0.305	0.58 - 6.92	SVR	Sph	-	
		Clay	Sample	WL		0.305		0.55 - 5.06	SVR	Sph	

		Clay	Sample	Preconsolidation stress, $\sigma_{\rm p}$		0.305		0.64 - 1.90	SVR	Sph	
		Clay	Sample	Friction angle, ϕ (°)		0.305		0.33 - 2.01	SVR	Sph	
		Clay	Sample	PL		0.305		0.55 - 4.42	SVR	Sph	
		Clay	Sample	Su		0.305		0.82 - 1.43	SVR	Sph	
		Clay	Sample	Wn		0.305		0.91 - 7.10	SVR	Sph	
Longview, WA, USA		Silt	CPT	q _t	10	0.05		0.11 - 0.46	MM	Varies	Stuedlein (2011)
Baytown, TX,		Medium Stiff to Stiff Clay	CPT	q _t	9	0.02	4.0 to 9.90	0.16 - 0.48	MM	Varies	Studlein et al. (2012)
USA		Stiff to Very Stiff Clay	СРТ	qt	3	0.02	2.97 - 9.20	0.54 - 1.17	MM	Varies	Stuediem et al. (2012)
Hollywood,	Beach Deposits	Clean and silty	СРТ	q_t	25	0.02	1.6 - 6.7	0.33 - 0.78	мм	Varias	Bong and Stuedlein
SC, USA	Beach Deposits	sand	CFT	$\mathbf{f}_{\mathbf{s}}$	25	0.02	1.6 - 6.7	0.26 - 0.98	IVIIVI	varies	(2017)
Hollywood,	Ranch Danosita	Clean and silty	СРТ	q_{c1N}	25	0.02	1.6 - 6.7	0.35 - 0.67	MM	Varias	Bong and Stuedlein
SC, USA	Beach Deposits	sand	CFT	q _{c1Ncs}	25	0.02	1.6 - 5.6	0.28 - 0.82	IVIIVI	varies	(2018)
Taranto, Italy		Clayey silt to silty clay: upper clay	СРТ	qt	5			0.20 - 0.44	MM	CExp	Cafaro & Cherubini
		Clayey silt to silty clay: lower		q _t	5			0.19 - 0.72			(2002)
		Sand		q _t	7	0.05	7.86 – 13.65	0.63 - 0.77	ML, MM		
Wufeng,		Sand		F_R	7	0.05	5.36 - 9.13	0.22 - 0.47	ML, MM		$\mathbf{Y}_{120} \neq 21 (2018)$
Taiwan		Clay		q _t	7	0.05	11.19 – 24.99	0.36 - 0.40	ML, MM		Ald0 et al. (2018)
		Clay		F_R	7	0.05	11.16 – 19.55	0.15 - 0.16	ML, MM		
Oakland, California, USA	-	Silt mixtures	СРТ	q _{c1N}	2	0.05	-	0.82-0.96	ММ	Mainly SMK; also SExp, and	Uzielli et al. (2005) ¹

¹ The soil types refer to the classification by Robertson (1990). The different number of soundings among the two parameters, at each location and for each soil type class, depends on cases where Autocorrelation models were not applicable or the CPT profiles were non-stationary.

										QExp
Oakland, California, USA	-	Silt mixtures	СРТ	F _R	1	0.05	-	0.49	MM	Mainly CExp; also SExp
Oakland, California, USA	_	Sand mixtures	СРТ	F _R	1	0.05	-	0.60	MM	CExp
Oakland, California, USA	-	Clean sand	СРТ	q _{c1N}	3	0.05	-	0.68-1.11	ММ	Mainly QExp; also SMK, CExp, and SExp
Oakland, California, USA	-	Clean sand	СРТ	F _R	1	0.05	-	0.60	ММ	Mainly SMK; also CExp, and SExp
Mid-America earthquake regions, USA	-	Silt mixtures	СРТ	q _{c1N}	2	0.05	-	0.33-0.73	ММ	Mainly SMK; also SExp and QExp
Mid-America earthquake regions, USA	-	Silt mixtures	СРТ	F _R	1	0.05	-	0.40	MM	Mainly CExp; also SExp
Mid-America earthquake regions, USA	-	Clean sand	СРТ	q _{c1N}	19	0.05	-	0.39-0.97	ММ	Mainly QExp; also SMK, CExp, and SExp
Mid-America earthquake regions, USA	-	Clean sand	СРТ	F _R	11	0.05	-	0.28-0.59	MM	Mainly SMK; also

										CExp, and SExp
Texas, USA	-	Silt mixtures	СРТ	qcin	1	0.05	-	0.99	ММ	Mainly SMK; also SExp, and QExp
Texas, USA	-	Silt mixtures	СРТ	F _R	2	0.05	-	0.12-0.13	ММ	Mainly CExp; also SExp
Texas, USA	-	Sand mixtures	CPT	q _{c1N}	1	0.05	-	0.35	MM	SMK
Adapazari, Turkey	_	Clay, silty clay	СРТ	qcin	5	0.05	-	0.28-0.64	ММ	Mainly CExp; also SMK, SExp, and QExp
Adapazari, Turkey	-	Clay, silty clay	СРТ	F _R	3	0.05	-	0.26-0.45	ММ	Mainly SExp; also SMK, and CExp
Adapazari, Turkey	-	Clean sand	СРТ	qcin	3	0.05	-	0.39-0.79	ММ	Mainly QExp also SMK, CExp and SExp
Adapazari, Turkey	-	Clean sand	СРТ	F _R	2	0.05	-	0.19-0.26	ММ	Mainly SMK; also CExp, and SExp

Treasure Island, San Francisco Bay, California, USA-	-	Clay, silty clay	СРТ	qcin	4	0.05	-	0.13-0.23	ММ	Mainly CExp; also SMK, SExp and QExp	
Treasure Island, San Francisco Bay, California, USA	-	Clay, silty clay	СРТ	F _R	3	0.05	-	0.13-0.28	ММ	Mainly SExp; also SMK, and CExp	
Konaseema, India	-	Stiff clay	СРТ	qc	1	0.05	-	0.85	MM	SExp	Haldar and Sivakumar Babu (2009)
Yuanlin, Taiwan	-	Sandy and clayey layers	СРТ	q _c	71	0.05	-	0.1-3.9	MM	SExp	Live and Chan (2010)
Yuanlin, Taiwan	-	Sandy and clayey layers	CPT	f_s	71	0.05	-	0.2-1.9	MM	SExp	Liu and Chen (2010)
Tarsuit P-45 island, Canadian Beaufort Sea, Canada	-	Sand	СРТ	qc	18	0.05	1.69-13.69	0.42-0.44	ММ	SExp	Lloret-Cabot et al. (2014)
Urmia, Iran	-	Clay and organic clay, with silty and sandy inclusions	СРТ	qc	8	0.02-0.1	-	0.21-2.33	ММ	_	Jamshidi Chenari and Kamyab Farahbakhsh (2015)
Central Europe	-	Lignite	CPT	q_c	42	0.05	-	0.44-0.56	MM	SExp	
Central Europe	-	Lignite	CPT	f_s	42	0.05	-	0.36-0.45	MM	SExp	Bagiliska et al. (2010)
Taranto, Italy	-	Yellow clay	СРТ	q _c	15	0.05	-	0.195-0.436	MM	SExp	Kowo and Dula (2020)
Taranto, Italy	-	Blue-gray clay	CPT	q_c	15	0.05	-	0.185-0.720	MM	SExp	Kawa aliu Fula (2020)
Bologna district, Italy	-	Silt and sand mixtures	CPT	q _c	182	0.05	1400-11000	2-30	SVR	Sph	
Bologna district, Italy	-	Silt and sand mixtures	CPT	f _s	182	0.05	6000	2-15	SVR	SExp (x); Sph(y)	Vessia et al. (2020)
Bologna		Clay silt	CPT	q_c	7	0.05	-	0.13-1.03	MM	QExp,	Pieczyńska-Kozłowska

district, Italy									SExp	et al. (2017)
Brindisi, Italy	Silt mixture (silty clay and clayey silt)	СРТ	Su	6	0.02		1.6	MM	SExp	Cherubini and Vessia (2005)
Taranto, Italy	Stiff overconsolidated Taranto clay	СРТ	qc	15			0.2-0.4 (upper brownish-yellow layer) 0.2-0.7 (lower grey layer)	ММ		Cherubini et al. (2007)
Fivizzano Italy	Alluvial	Down Hole	Su	2			1.4-2.6			Cherubini et al. (2007)
TTVIZZano, Itary	Deposits	Down Hole	\mathbf{S}_{u}	2			1.8 -2.0			Cherubhin et al. (2007)
	Ankara Clay		w _L				4-6.2			
Antrono Tuntrov	Ankara Clay		Wn				2.5-5.5			Akbas and Kulhawy
Alikara, Turkey	Ankara Clay		Su				3-Jan			(2010)
	Ankara Clay	SPT	N value				3-3.8			
NGES at Taxas	Sand	CPT	q _c , fs			2-25, 7-19	0.26-3.14	MM		Akkawa and
A&M, USA	Clay	СРТ	q _c , fs			2.5–30, 2–14	0.3-3.62	MM		Vanmarcke (2003)
Karameh Dam, Jordan	-		Su			2-10				Al-Homoud and Tanash (2001)
	Clean sand and sand fill	SPT	N value				0.3-4			Alonso and Krizek (1975), Lumb (1975), reported by Huber (2013)
	Organic soft clay	VST	Su			-	1.2			Asaoka and Grivas
	Organic soft clay	VST	Su			-	3.1			(1981)
New York	Soft clay, New York	VST	Su				2.5-6		SExp	Asoaka et al (1981)
	Sensitive clay	VST	$\mathbf{S}_{\mathbf{u}}$			46	2			Baecher (1982)
Bełchatów (Central Poland)	Lignite mine waste dump	СРТ	qc	4		0.8-3.5	0.15-0.22		SExp	Baginska et al. (2018)
		СРТ	q _c			Isotropic	0.36-0.56		SExp	Baginska et al. (2016)

									QExp CExp	
Bangkok, Thailand	Very soft clay	VST	$\mathbf{S}_{\mathbf{u}}$	41	1	22.1	1.1			Bergado et al. (1994)
	Sand and clay	СРТ	qt	333	0.01/0.02	12.2-16.1	0.07-0.78	MM	SExp SMK QExp	
	Marine Clay	CPT	q _c				0.78		SExp	
	Marine Sand	CPT	q_c				0.08		SMK	
	Continental Clay	CPT	q_c				0.21		SMK	
Pearl River	Marine Alluvial Clay	CPT	q _c			12.15	0.5		SExp	Bombasaro and Kasper
Estuary, China	Marine Alluvial clay with sand laminae	СРТ	qc			15.67	0.29		SExp	(2016)
	Marine Alluvial sand	СРТ	q_c			15	0.07		SExp	
	Fluvial alluvial clay	CPT	q _c			15.06	0.08		SExp	
	Fluvial alluvial sand	CPT	q _c			16.11	0.38		SExp	
Jijel port, Algeria	Onshore sandy soils (loose to medium dense sands, dense fine sands and silty sands)	UC and DPL	qc	10	0.02		0.32-1.32 (0.78)			Bouayad (2017)
Taranto, Italy	Taranto clay	CPT	q _c	15	0.2		0.195-0.72		CExp	Cafaro and Cherubini (2002)
Suqian City, China	Alluvial deposit	CPT	q _{c1N}	16	0.05	1.1-1.5	0.2-0.29	MM		Cai et al. (2016)
Deltaic Soils, Canada	Sand	CPT	q _c			0.025	0.13-0.71			Campanella et al. (1987)
Lian-Yun-Gang City, China	Undrained engineered slope	UC or VST	S_u				1.04	APM		Chai et al. (2002)
Saint-Hilaire, Canada		CPT, VST		16 CPT 27 VST	CPT: 0.02 VST: 0.5		1.5	SVR	Sph	Chiasson et al. (1995)
	Sandy soil	CPT	q _c				0.1-1.0		QExp	Cheng et al. (2000)

	Clay	CPT	q_{c}				0.1-1.8		CExp	
	Soft clay	СРТ	q _c				0.2-2.0		Bin	
Gulf of Mexico	Offshore soils	Sample	Su	16		9000	7.1 - 9.1			Cheon and Gilbert (2014)
	Clay	CPT					042-0.96			Cherubini et al. (2016)
Montreal, Canada	Sensitiv clay	VST	Su	27	X: 10m Y: 0.5m		4			Chiasson et al. (1995)
	Clay	VST	$\mathbf{S}_{\mathbf{u}}$			46–60	2.0-6.2		SExp	Ching et al. (2011)
	Undrained engineered slope	UC or VST	Su				2.3	APM		Dascal and Tournier (1975)
James Bay, Quebec, Canada	Sensitive clay	VST	S_u	35						DeGroot and Baecher (1993)
	Undrained engineered slope	UC or VST	Su				1.2	APM		Eide and Holmberg (1977)
Jutland, Denmark	Clayey silty sand	СРТ	qcin	21	0.02		0.2-0.5	MM		Firouzianbandpey et al. (2014)
	Undrained engineered slope	UC or VST	$\mathbf{S}_{\mathbf{u}}$				0.62	APM		
	Undrained engineered slope	UC or VST	$\mathbf{S}_{\mathbf{u}}$				1	APM		
	Undrained engineered slope	UC or VST	Su				0.6	APM		
	Undrained engineered slope	UC or VST	$\mathbf{S}_{\mathbf{u}}$				1.1	APM		Flaate and Preber (1974)
	Undrained engineered slope	UC or VST	Su				1	APM		
	Undrained engineered slope	UC or VST	Su				1.8	APM		
	Undrained engineered slope	UC or VST	$\mathbf{S}_{\mathbf{u}}$				1.25	APM		
	Undrained engineered slope	UC or VST	Su				0.72	APM		Flaate and Preber(1974), La Rochelle et al.(1974)
	Shanghai silty clay						0.31-0.42		SExp CExp	Haldar and Sivakumar
Konaseema site, India	Silty clay	СРТ	q _c	1	0.2		0.8–6.1		SExp	Babu (2009)
	Undrained	UC or	$\mathbf{S}_{\mathbf{u}}$				2.5	APM		Hanzawa (1983),

	engineered slope	VST UC or								Kishida et al. (1983), Hanzawa et al. (1980),
	engineered slope	VST	Su				2.5	APM		Hanzawa (1983) , Hanzawa et al. (1994)
	Undrained engineered slope	UC or VST	S_u				0.57	APM		Hallzawa et al. (1994)
Cape Cod, Massachus, USA			k	16		2-10	0.2-1	SVR	SExp	Hess et al. (1992)
	Clay						0.25–2.5		SExp	Hicks and Samy (2002)
	Offshore soils	CPT	q _c			30				Hoeg (1977): Tang
	Marine clay (different levels)	СРТ	q _c			35-60				(1979)
Tokyo, Japan	Soft clay	UCC	Su	5	1 or 2	40	2			Honjo and Kuroda
Tokyo, Japan				5		80	4		SExp	(1991)
	Clay		σ_t			1.22	1.22		SExp	Hsu and Nelson (2006)
Emme Valley, Berne, Switzerland			k	16		15-20	0.63			Hufschmied (1986)
Chicago, USA	Undrained engineered slope	UC or VST	\mathbf{S}_{u}	8			1.6	APM		Ireland (1954)
Adelaide, Australia	Clay	СРТ	q _c	200	0.005	0.14	0.06-0.25 (mean 0.148)	MM	SExp	Jaksa (1995)
South Parlands, Adelaide, Australia	Relatively homogenous, stiff, overconsolidated clay known as Keswich Clay	СРТ	qc	30	0.005		0.63-2.55			Jaksa et al. (1999)
Urmia, Iran	clean sand, clay	СРТ	q _c , fs	8	0.1 or 0.02		0.21-2.33		SExp	Jamshidi Chenari and Kamyab Farahbakhsh (2015)
	In situ soils					30–60	1.0–6.0		SExp	Ji et al. (2012)
	Offshore sand	CPT	q _c			14-38	0.66-0.99			
	Offshore soils	CPT				24.6-66.5				
	Offshore soils	Sample	Su				0.48-7.14			Keaveny et al. (1989)
	Offshore soils	СРТ	q _c			14-38				
	Offshore	Sample	Su				0.6699			

	cohesive soil									
	Offshore soil	UU	$\mathbf{S}_{\mathbf{u}}$			-	3.6			
	Offshore soil	DST	$\mathbf{S}_{\mathbf{u}}$			-	1.4			
	Clean sand	СРТ	q _c			-	1.6			Kulatilake and Ghosh (1988)
	Silty Clay	СРТ		28		5-12	1.4-2			Lacasse and de Lambellerie (1995)
	Offshore sand	СРТ	q_{c}			25-67c		MM	SExp	T 1 N 1'
	Laminated clay	CPT	q_{c}			9.6	-			Lacasse and Nadim (1996)
	Dense sand	CPT	q_{c}			37.5	-			(1)))
Portsmouth, N.H., USA	Undrained engineered slope	UC or VST	Su	3			0.94	APM		Ladd (1972)
	Undrained engineered slope	UC or VST	Su	3			2.5	APM		Lafleur et al. (1988)
	Taiyuan silty clay	DST				36.2–41.7	0.37–0.58		Bin	
	Taiyuan silty clay	DST				36–41.4	0.35–0.49		Bin	
	Taiyuan silt	DST				41.5–45.1	0.6–0.84		Bin	
	Taiyuan silt	DST				41.8–45.5	0.54-0.92		Bin	L_{i} at al. (2002)
	Hangzhou silty clay	DST				40.5-45.4	0.52–0.75		Bin	Li et al. (2005)
	Hangzhou silty clay	DST				40.4–45.2	0.49–0.71		Bin	
	Hangzhou clay	DST					0.5 - 0.77		Bin	
	Hangzhou clay	DST					0.59–0.73		Bin	
	Clay-bound sand	СРТ	q _c			6-13	0.34-1.7	MM	SExp Sph	Lingwanda et al. (2017)
Yuanlin,	Sand, silt, and clay	СРТ	q _c , fs	71	0.05	126.9-163.9		MM	SExp	Liu and Chan (2006)
Taiwan	Sand, silt, and clay	СРТ	q _c , fs	/1	0.03	66-1546	0.18-1.96	MM	SExp	Liu and Chen (2000)
Yuanlin, Taiwan	onshore alluvial deposits (loose sandy soils, cohesive soils, medium dense to	СРТ	qc	71	0.05	62-2000	1.72-2.53		SExp	Liu and Chen (2010)

	dense sands and									
	Offshore clays	СРТ	q _c				0.05-1			Liu et al. (2015)
Tarsuit P-45, Canada	Filled sand in artificial Island	СРТ	q _c	18		1.7–15.9	0.4	MM		Lloret-Cabot et al. (2014)
	Marine clay, Japan						1.3-2.7		SExp	Matsuo (1976)
						50-70		SVR	Sph	
						40-60		SVR	Sph	
						40-70		SVR	Sph	Mulla (1988)
						60-80		SVR	Sph	
						40-60		SVR	Sph	
Veda, Sweden	Clay	СРТ	qt	16		20	0.4	MM		Müller et al. (2014)
CDP1 Platform, North Sea	Different soil units	СРТ	q _c	39			0.18-0.39			Nadim (2015)
	Clay and silt clay	СРТ	q _c			283, 225		MM		Ng and Zhou (2010)
	Yan'an silty clay						1.47			
	Yan'an silty clay						1.44			
	Jiangzhang silty clay						6.47			N:
	Jiangzhang silty clay						2.96			M et al. (2002)
	Tongguan silt						7.19			
	Tongguan silt						1.2			
NGES-UH, USA	Alluvial deposit	СРТ	q _c	A:12 B: 44	0.15	2-7	1-2	SVR		O'Neill and Yoon (2003)
	onshore alluvial deposits (loose sandy soils, cohesive soils, medium dense to dense sands and clay layers)	СРТ	fs				0.18-1.96			Oguz et al. (2019)
	Clayey silty sand	СРТ	fs				0.2			
North Sea	Offshore sand	СРТ		18			0.4-2.9	ML		Overgård (2015)

	and clay sublayers								
	Onshore two clay sites						0.11-0.29		Pantelids and Christodoulou (2017)
	Clay		Su	5			0.8-6.1 (2.5)		
	Sand, Clay	CPT	q _c	7			0.1-2.2 (0.9)		Phoon and Kulhawy
	Clay	CPT	qt	10			0.2-0.5 (0.3)		(1999a),Phoon and
	Clay	VST	Su	6		46-60	2-6.2		Kulhawy (1999b)
	Clay, loam		Wn	3			1.6-12.7 (5.7)		
	Clay	СРТ	Su	5			0.8-6.1(mean 2,5)		
	Clay	CPT	q _t	x: 2 z: 7		23-66 (mean 44.5)	0.1-2.2 (mean 0.9)		Phoon et al. (1005)
	Sand and clay	СРТ	q_{c}	x:11 z: 10		3-80 (mean 47.9)	0.2-0.5 (mean 0.3)		Phoon et al. (1995)
	Clay	VST	S_u	x: 3 z: 6		46-60(mean 50.7)	2-6.2 (mean 3.8)		
Treasure Island Naval Station, California, USA	Offshore sediments	CPT, Lab tests	Su		0.02		0.38-0.8		Phoon et al. (2003)
	Undrained engineered slope	UC or VST	Su				0.96-2.7	APM	Pilot (1972), Pilot et al. (1982), Talesnick and Baker (1984)
	Sand	SPT	N value			12.1	0.95		Popescu et al. (1995)
	-		Su			Isotropic 0.5. rarely more than 10			Rackwitz (2000)
	Undrained engineered slope	UC or VST	S_u				2.5	APM	Ramalho-Ortigão et al.
	Undrained engineered slope	UC or VST	Su				1.8	APM	Ferkh and Fell (1994)
Columbs, Mississippi, USA			k			12.7	1.6		Rehfeldt, Gelhar, et al. (1989),Rehfeldt, Hufschmied, et al. (1989), Young and Boggs (1990)

				58	0.15	7.5-22.6	1-2.3		SExp	
				58	0.15	25-50	1.5-3		SExp	Rehfeldt et al. (1992)
	Clay	DST				92.4	1.2-2		QExp	Ronold (1990)
						750		SVR	Sph	Rosenbaum (1987)
	Clay	CPT	q _c			10-62	1.3–4.0		SExp	Salgado and Kim
	Sand	CPT	q _c			35–75	2.2-3.0		SExp	(2014)
Lodalen, Oslo, Norway	Undrained engineered slope		Su	14			1.8	APM		Sevaldson (1956)
	Very soft clay (sand inclusion)	UC or VST					0.16-0.32 (0.23)			
	Mud and very soft clay	СРТ					0.14-1 (0.37)		SExp QExp CExp	Huwang and Linping
	very soft clay and clay	СРТ					0.16-0.57 (0.37)			(2015)
	Clay	CPT					0.13-0.32 (0.24)			
	Silty Clay	CPT	q _c				0.1-0.43 (0.23)			
		CPT				13-19	3	SVR	SExp	
	Marine Clay		$\mathbf{S}_{\mathbf{u}}$				6			Soulie' et al. (1990)
	Sensitive clay		Su			30	3			
Canadian Forces Base Borden, Ontario, Canada		VST	k		0.2-0.3	2.8	0.12			Sudicky (1986), Freyberg (1986)
						55			QExp	T (1070)
		СРТ				35-60			QExp	Tang (1979)
		СРТ				5.68-9.27		SVR		
						8.89-20		SVR		Unlu et al. (1990)
						4.02-7.5		SVR		
Turkey and	Sand, Clay, Silt (Mixture)		qc	40	0.05		0.13-1.11 (0.7)			Uzialli et al. (2005)
North America	Sand, Clay, Silt (Mixture)	СРТ	F _R	25	0.05		0.12-0.6 (0.36)			021em et al. (2003)
	Superficial soft	CPT	Soil layer			22.2				Valdez-Llamas et al.

	clay		thickness							(2003)
	Superficial soft clay		Wn				0.8-2.0			
	Deep deposits with alternating clayey and sandy soils		Wn			1000	21			
	New Liskeard varved clay		S _u , N value	25		46	5			Vanmarcke (1977)
Leidschendam, Netherlands	Sand		qz	18	2-5	22-34			SExp	Vrouwenvelder and Calle (2003)
Deltaic Soils, Canada	Sand	СРТ	q _c		0.025		0.24-0.32			Wickremesinghe and Campanella (1993)
	Undrained engineered slope	СРТ	Su				0.3	APM		Wilkes (1972)
	Chicago clay	UC or VST	S_u			-	0.4			Wu (1974)
North Sea	Fine sand	UC	q _c	24		26	0.4	MM	SExp	Wu et al. (1987)
	Clay	CPT				10–40	0.5–3.0		SExp	
	Alluvial soil					30–49	0.2–0.9		SExp	
	Ocean and lake sedimentary soils					40-80	1.3-8.0			Wu et al. (2011)
	Moraine soil						2			
	Aeolian soil						1.2–7.2			
	Undrained engineered slope		Su				1.5	APM		Wu et al. (1977)
	Chicago clay	UC or VST					0.79–1.25		SExp	Via (2000)
	Saturated clay, Japan						1.25–2.86			Ale (2009)
	Tianjin port clay		q _c			8.37	0.132-0.322			
	Tianjin port silty clay	СРТ	q _c			9.65	0.095–0.426			Yan et al. (2009)
	Tianjin port silt	СРТ	q _c			12.7	0.140-1.0			
	Sandy	СРТ	N value	3			1.36-3.01		SExp QExp	Zhang and Chen (2012)
	Sand, Clay, Silt	SPT	q _c			-	0.36-4.92	MM	-	Sasanian et al. (2018)

		(Mixture)									
Adapazari area, Turkey		Sand, Clay, Silt (Mixture)	СРТ	q _c	1	0.02	-	0.16	VRF	-	Pishgah and Chenari (2013)
		Sand, Clay, Silt (Mixture)	СРТ	qc			-	0.44-1.52	MM, VRF	SExp	Eslami Kenarsari et al. (2013)
B.C., Canada		Sand, Clay, Silt (Mixture)	СРТ	qc	1		-	0.5		SExp	Jamshidi Chenari et al. (2018)
Canadian Beaufort sea shelf		(0-3 m below sea bottom)	СРТ	q _c	6	9	55				Zhang et al. (2016)
			СРТ	q _c				0.6-1.4			
			СРТ	q _c			7	0.59-1.44	APM		
			CPT	q _c				0.4-1.7			
		Fine to medium	CPT	q _c				0.35-1.5	MM	Sph	
		grained kaolinic	CPT	q _c				0.34-1.4	MM	SExp	
		sandstones	CPT	q _c			10		MM	Tri	D
		overlain by consolidated	CPT	N_{10}				0.3-1.8			Prastings (2019)
		clay-bound	DPL	N_{10}			5	0.54-0.98	APM		
		sands	DPL	N_{10}				0.38-1.5	MM	Sph	
			DPL	N_{10}				0.37-1.59	MM	SExp	
			DPL	N ₆₀			7	1.7-4.2	APM		
			SPT	М			7	2.5-2.9	APM		
Japan	Marine bay	Clay, Silty Clay	UC	Su	1	0.25-0.5		1.2-2.5	MM	SExp	Matsuo and Asaoka (1977)
Tokyo Japan	Marine bay	Clay	UC	Su	1	231	660		MM	SExp	N (1004)
Hokkaido Japan	Alluvial Layer	Organic clay	UC	Su	1	36	105		MM	SExp	Matsuo (1984)
Japan	Alluvial Layer	Sand	СРТ	qt	3	0.05		0.3-0.5	MM	SExp	Honjo and Otake (2012)
Okayama Japan	Embankment	Silty sand	СРТ	q_t	10	X: 5 Y: 0.05	20	1.02	ML	SExp	
Kyoto Japan	Embankment	Silty sand	СРТ	q _t	11	X: 5 Y: 0.05	20	0.92	ML	SExp	maide, et al. (2015)
Hiroshima Japan	Embankment	Silty sand	СРТ	log (N value)	8	X: 2.0 Y: 0.05	4.8	0.58	MM	SExp	Imaide, et al. (2019)

Okayama Japan	Embankment, Alluvial layer	Silty sand, Clay, Sand	СРТ	log (N value)	24	X: 5 Y: 0.05	20	0.92	ML	SExp	Nishimura et al. (2017)
Okayama Japan	Embankment	Silty sand	SWS	N value	15	X: 2 - 5 Y: 0.25	54.2	4.06	MM	SExp	Nishimura et al. (2016)
Okayama Japan	Embankment	Silty sand	SWS	N value	9	X: 5 Y: 0.25	30.8	1.20	MM	SExp	Nishimura et al. (2010)

Table 3.3. Summary of the coefficient of inherent variability reported in the literature

Site Location	Geomorphology	Soil type	Type of Measurement	Parameter	# soundings	Data interval (m)	COV _h (%)	COV _z (%)	Method	Model	Reference
		Clay		Cohesion, C (kPa)				9-16			
Jiangsu,		Clay		Friction angel, ϕ (°)				16-73			Zhang et al.
China		Clay		Young's modulus, E (MPa)				14-21			(2009)
		Silty clay		Natural moisture content, w (%)	49 sets (including 2695 samples)			2-18			
		Clay		Natural moisture content, w (%)	15 sets (including 1185 samples)			3-15			
		Silty sand		Natural moisture content, w (%)	22 sets (including 550 samples)			3-17			
		Silty clay		Liquid limit, w_L (%)	42 sets (including 2100 samples)			4-18			
		Clay		Liquid limit, w_L (%)	12 sets (including 996 samples)			1-15			
Jiangsu,		Silty clay		Unit weight, γ	50 sets (including 2900 samples)			1-5			Zhang et al.
China		Clay		Unit weight, γ	15 sets (including 1200 samples)			1-4			(2010)
		Silty sand		Unit weight, γ	21 sets (including 504 samples)			1-4			
		Silty clay		Plasticity index, PI	14 sets (including 364 samples)			9-38			
		Silty clay		Liquidity index, LI	14 sets (including 364 samples)			11-47			
		Silty clay		Void ratio, e	16 sets (including 512 samples)			5-15]		
		Silty sand		Void ratio, e	14 sets (including 252 samples)			6-19			

	Silty clay		Young's modulus, E (MPa)	30 sets (including 960 samples)			5-48			
	Clay		Young's modulus, E (MPa)	7 sets (including 427 samples)			12-34			
	Silty sand		Young's modulus, E (MPa)	12 sets (including 252 samples)			15-51			
	Silty clay		Cohesion, C (kPa)	43 sets (including 1419 samples)			7-51			
	Clay		Cohesion, C (kPa)	12 sets (including 540 samples)			13-44			
	Silty sand		Cohesion, C (kPa)	12 sets (including 192 samples)			23-106			
	Silty clay		Friction angle, ϕ (°)	43 sets (including 1419 samples)			7-54			
	Clay		Friction angle, ϕ (°)	12 sets (including 540 samples)			7-28			
	Silty sand		Friction angle, ϕ (°)	12 sets (including 192 samples)			7-31			
			Natural moisture content, w (%)	656 samples			4.2-37			
			Plasticity index, PI	153 samples			15-36			
			Clay content (<2µm)	143 samples			7.8-33			
Paris,	Ypresian	CD and CU triaxial tests	Friction angel, ϕ (°)	91 samples			23.8-24.1			Khadija et al.
France	plastic clay	CD and CU triaxial tests	Cohesion, C (kPa)	65 samples			13-38			(2020)
			Young's modulus, E (MPa)	123 samples			59-68			
		Oedometer test	Coefficient of earth pressure at rest, K ₀	70 samples			6.8-33			
College Station, TX, USA	Clay	СРТ	qc	6	0.02		5 - 36	ММ	SExp	Kulatilake & Um (2003)
College	Sand site: Silty Sand	СРТ	q _c	22	0.02		32 - 69			A 1-1-2
Station, TX,	Sand site: Clean Sand	СРТ	q _c	22	0.02]	15 – 75	MM	SExp	Akkaya & Vanmarcke
USA	Sand site: Clayey	СРТ	qc	22	0.02		33 - 70			(2003)

		Sand									
		Sand site:	СРТ								
		Hard Clay		q_{c}	22	0.02		4 - 53			
		Clay site:	СРТ								
		very stiff		q_c	24	0.02		30 - 66			
		Clay	CDT								
		Clay site:	CPI	q_c	24	0.02		7 – 31			
		Clay site:	СРТ								
		very stiff	011	q _c	24	0.02		48 - 100			
		clay									
		Clay site:	CPT	a	24	0.02		15 - 72			
		hard clay		Ч¢	27	0.02		15 - 72			
		Sand site:	CPT	fs	22	0.02		44 - 82			
		Silty Sand	CDT	5				_			
		Sand site: Clean Sand	CPT	t _s	22	0.02		18 - 67			
		Sand site:	СРТ	fs							
		Clayey	011	-5	22	0.02		34 - 71			
		Sand									
		Sand site:	CPT	\mathbf{f}_{s}	22	0.02		8 - 56			
		Hard Clay	~~~~	-	22	0.02		0 50			
		Clay site:	CPT	f _s	24	0.02		20 59			
		very sum			24	0.02		50 - 58			
		Clay site:	СРТ	f.							
		sand	011	15	24	0.02		15 - 44			
		Clay site:	СРТ	fs							
		very stiff			24	0.02		39 - 82			
		clay									
		Clay site:	CPT	$\mathbf{f}_{\mathbf{s}}$	24	0.02		15 - 60			
Lanaria		hard clay									
Longvie w WA	Alluvial	NC Silt	СРТ	a	10	0.05		8 2 24 1	MM	Varias	Stuedlein
W, WA, USA	deposits	NC SII	CLI	q t	10	0.05		0.2 - 24.1	101101	varies	(2011)
0.011		Medium					12	16.1			
Baytown	Marine deposits	Stiff to	CPT	q_t	9	0.02	13 -	10.1 - 76 8			Stuadlain at
, TX,		Stiff Clay					55	/0.0	MM	Varies	al (2012)
USA	Marine Deposits	Stiff to	СРТ	(]t	3	0.02	9 - 14	7.8 - 10.8			ul. (2012)
	in anne Deposito	Very Stiff	<u> </u>	<u>א</u> י	5	0.02	/ 11	10.0			

		Clay									
Hollywo	Beach Deposits	Clean and	СРТ	q _t	25	0.02	9 - 80	37.8 – 55.8	MM	Varies	Bong and Stuedlein
USA	Deach Deposits	silty sand	CIT	f_s	25	0.02	17 – 66	17.2 – 70.2	101101	varies	(2017)
Hollywo	Baash Danasita	Clean and	СРТ	Q c1N	25	0.02	9 - 81	34.8 – 52.1	MM	Varias	Bong and
USA	beach Deposits	silty sand		q_{c1Ncs}	25	0.02	4.2 - 35	11.1 – 38.0	IVIIVI	varies	(2018)
Oakland , Californi a, USA	-	Silt mixtures	СРТ	q _{c1N}	2	0.05	-	18-20	MM	Mainly SMK; also SExp, and QExp	
Oakland , Californi a, USA	-	Silt mixtures	СРТ	F _R	1	0.05	-	17	ММ	Mainly CExp; also SExp	
Oakland , Californi a, USA	-	Sand mixtures	CPT	F _R	1	0.05	-	38	ММ	CExp	Uzielli et el
Oakland , Californi a, USA	-	Clean sand	СРТ	q _{c1N}	3	0.05	-	18-25	ММ	Mainly QExp; also SMK, CExp, and SExp	(2005) ²
Oakland , Californi a, USA	-	Clean sand	СРТ	F _R	1	0.05	-	22	MM	Mainly SMK; also CExp, and SExp	
Mid-Am erica	-	Silt mixtures	СРТ	q _{c1N}	2	0.05	-	19-28	MM	Mainly SMK;	

 $^{^2}$ COV_z values refer to the Inherent Coefficient of Variation, calculated after the vertical trend removal.

earthqua ke regions, USA										also SExp, and QExp
Mid-Am erica earthqua ke regions, USA	-	Silt mixtures	СРТ	F _R	1	0.05	-	18	ММ	Mainly CExp; also SExp
Mid-Am erica earthqua ke regions, USA	-	Clean sand	СРТ	Q _{c1N}	19	0.05	-	17-38	ММ	Mainly QExp; also SMK, CExp, and SExp
Mid-Am erica earthqua ke regions, USA	-	Clean sand	СРТ	F _R	11	0.05	-	10-31	ММ	Mainly Secon d-orde r Marko v; also CExp, and SExp
Texas, USA	-	Silt mixtures	СРТ	q _{c1N}	1	0.05	-	33	ММ	Mainly SMK; also SExp, and QExp
Texas, USA	-	Silt mixtures	СРТ	F _R	2	0.05	-	13-15	ММ	Mainly CExp; also SExp
Texas, USA	-	Sand mixtures	СРТ	q _{c1N}	1	0.05	-	24	MM	SMK
Adapaza ri, Turkey	-	Clay, silty clay	СРТ	q _{c1N}	5	0.05	-	11-21	MM	Mainly CExp; also

										SMK,	
										SExp,	
										and	
										QExp	
										Mainly	
Adapaza		~								SExp;	
ri.	-	Clay, silty	CPT	FR	3	0.05	-	14-21	MM	also	
Turkey		clay								SMK,	
5										and	
										CExp	
										Mainly	
										QExp;	
Adapaza		CI I	CDT			0.05		20.24	107	also	
rı,	-	Clean sand	CPT	q_{c1N}	3	0.05	-	20-24	MM	SMK,	
Turkey										CExp,	
										and	
										SExp	
										Mainly	
Adapaza										SMK;	
ri,	-	Clean sand	CPT	F _R	2	0.05	-	48-59	MM	CEur	
Turkey										CExp,	
-										and	
Transura										Moinly	
Island										CEve	
Island,										cExp,	
Francisc		Clay, silty	СРТ	(Law)	4	0.05		2.5	мм	SMK	
o Bay	-	clay	CFI	Y c1N	4	0.05	-	2-3	IVIIVI	SIVIX, SEvn	
Californi										and	
										OEvn	
Treasure										QLAP	
Island										Mainly	
San										SExp;	
Francisc	-	Clay, silty	CPT	FR	3	0.05	-	10-18	MM	also	
o Bay		clay	011	I K	5	0.05		10 10		SMK,	
Californi										and	
a, USA										CExp	
Konasee											Haldar and
ma,	-	Stiff clay	CPT	q _c	1	0.05	-	37	MM	SExp	Sivakumar
India		-		•							Babu (2009)

Central Europe	-	Lignite	СРТи	q_{c}	42	0.05	-	23.55-88. 16	MM	SExp	Baginska et
Central Europe	-	Lignite	CPTu	$\mathbf{f}_{\mathbf{s}}$	42	0.05	-	31.99-11 1.53	MM	SExp	al. (2016)
Bologna district, Italy		Clay silt	СРТ	Su	7	0.05		17-51	Detrend +Resid uals Analys es		Pieczyńska- Kozłowska et al. (2017)
			Undrained shear test	su	89			32.7			
			Atterberg Limits	LL	181			14.1			Cherubini
Matera, Italy		Matera blue clay	Atterberg Limits	PL	181			16.2			and Vessia (2005)
			Unit weight	γ	579			4.6			(/
			Natural water content	Wn	579			12.9			
		Silt	Atterberg limits	LL				21.8			
Brindisi,		mixture (silty clay	Atterberg limits	PL				20.7			Cherubini and Vessia
Italy		and clayey	Unit weight	γ				4.9			(2005)
		silt)	Natural water content	Wn				26.4			
Fivizzan		Alluvial	Down Holo	V_{SH}	2			7-9.2			Cherubini et
o, Italy		Deposits	Down Hole	V_P	2			6.8-7.9			al. (2007)
Japan	Marine bay	Clay, or Silty Clay	UC	\mathbf{S}_{u}	1	0.25-0. 5			ММ	SExp	Matsuo and Asaoka (1977)
Tokyo Japan	Marine bay	Clay	UC	\mathbf{S}_{u}	1	231			MM	SExp	Matsuo
Hokkaid o Japan	Alluvial Layer	Organic clay	UC	Su	1	36			ММ	SExp	(1984)
Japan	Alluvial Layer	Sand	СРТ	qt	3	0.05			MM	SExp	Honjo and Otake (2012)
Okayam a	Embankment	Silty sand	СРТ	qt	10	X: 5.0 Y: 0.05			ML	SExp	Imaide, et al. (2015)

Japan										
Kyoto Japan	Embankment	Silty sand	СРТ	qt	11	X: 5.0 Y: 0.05		ML	SExp	
Hiroshi ma Japan	Embankment	Silty sand	СРТ	log (N value)	8	X: 2.0 Y: 0.05		ММ	SExp	Imaide, et al. (2019)
Okayam a Japan	Embankment + Alluvial layer	Silty sand, Clay, Sand	СРТ	log (N value)	24	X: 5.0 Y: 0.05		ML	SExp	Nishimura et al. (2017)
Okayam a Japan	Embankment	Silty sand	SWS	N value	15	X: 2-5 Y: 0.25		ММ	SExp	Nishimura et al. (2016)

Notations

Type of measurement:

- CPT Cone/piezoncone penetration test
- VST Vane shear test
- UC Unconfined compression
- SPT Standard penetration test
- DST Direct shear test
- DPL Dynamic probing light
- SWS Surface wave seismology

Properties:

- q_c Cone tip resistance
- q_{c1N} Normalized cone tip resistance
- qt Corrected cone tip resistance
- f_s Sleeve friction
- F_R Friction ratio
- S_u Undrained shear strength
- k Hydraulic conductivity
- N value SPT-N value
- N₁₀ Number of blows per 100 mm of penetration for DPL
- N₆₀ Energy normalized SPT-N value
- w_n Natural water content
- w_L Liquid limit

- w_P Plastic limit
- Cc Compression index
- Cs Swell index
- LI Liquid index
- γ Unit weight
- e₀ Void ratio

Method:

- APM Approximate method proposed by Vanmarcke (1977)
- MM Autocorrelation function fitting (method of moments)
- ML Maximum likelihood
- VRF Variance reduction function fitting
- SVR Semivariogram

Autocorrelation model:

SExp:	Single exponential
SMK:	Second-order Markov
QExp:	Squared exponential
Bin:	Binary
CExp:	Cosine exponential
Sph:	Spherical
Tri:	Triangular

4. Statistics for geotechnical design model factors

Chong Tang and Richard Bathurst

4.1 Introduction

During the historical development of the mechanics of deformable solids, the problems in geotechnical engineering are often categorized into two distinct groups, namely elasticity and stability (e.g., Terzaghi 1943; Terzaghi and Peck. 1948; Chen 1975). The elasticity problems deal with stress or deformation of soil without failure, such as point stress beneath a footing or behind an earth retaining wall, deformation around a tunnel or an excavation, and settlement analysis. The stability problems are associated with the determination of a load that will cause the failure of soil, such as bearing capacity, passive and active earth pressure, and slope stability. In many design codes/manuals (e.g., CEN 2004; AASHTO 2017; JRA 2017; CSA 2019), elasticity is usually considered as a serviceability limit state (SLS), while stability is frequently considered as an ultimate limit state (ULS). For practical design convenience, many useable and oftentimes analytically tractable models (link between theory and practice) have been developed for elasticity and stability analyses of geotechnical structures. Because of the natural (or inherent) variability of geomaterial properties, assumptions and simplifications made by calculation models and the difficulty to model construction disturbance, the predicted response will deviate from the measured one (typically on the safe side). This deviation can be directly captured by a ratio of the measured value (X_m) (e.g., load test) to the calculated value (X_c) that is called a model factor M $(=X_m/X_c)$ (ISO 2015). This method is practical, familiar to engineers, and grounded realistically on a load test database. The quantity X could be a load, a resistance, or a displacement, etc.

The simplest way to characterize the model factor M is to calculate the mean (bias) and coefficient of variation (COV) (dispersion). The mean of M would provide an engineer with a sense of the hidden factor of safety (FS) that either adds or subtracts from the nominal global FS, depending on whether the calculation method is conservative (mean>1) or unconservative (mean<1) in the average sense. It should not be inferred that a calculation method is conservative or otherwise for a specific case because M takes a range of values in actuality (hence it is random) that may depend on the scenarios covered in the database used in calibration. This random nature is practically significant, because it implies that a calculation method can be unconservative when applied to a specific case even though the method is conservative on the average. Therefore, it is also necessary to consider the degree of scatter (dispersion) in M and to ensure probability of a measured value being lower than the calculated value is capped at a known value say p%. This idea is conceptually

similar to EN 1997-1:2004 (CEN 2004), 2.4.5.2 (11) which recommends a cautious estimation (or characteristic value) for a geotechnical design parameter can be "derived such that the calculated probability of a worse value governing the occurrence of the limit state under consideration is not greater than 5%." The model factor statistics (mean and COV) have been widely used to develop load and resistance factor design (LRFD) of foundations (e.g., Paikowsky et al. 2004, 2010; Salgado et al. 2011; Ng and Fazia 2012; Abu-Farsakh et al. 2013; AbdelSalam et al. 2015; Seo et al. 2015; Stark et al. 2017; Asem et al. 2018; Machairas et al. 2018; Asem and Gardoni 2019; Heidarie Golafzani et al. 2020; Petek et al. 2020; Tang and Phoon 2021). The Federal Highway Administration mandated the use of LRFD for all federally funded new bridges after September 2007 (e.g., Brown et al. 2018; Hannigan et al. 2016; AASHTO 2017).

4.2 Summary Table

Table 4.1 summarizes previous studies on the characterization of model factors – mean (bias) and COV (dispersion). The dataset used in calibration include laboratory (scaled model or prototype in a centrifuge facility) (representing controlled soil condition) or in situ (representing natural soil condition) load tests. The results cover various geotechnical structures (e.g., shallow foundations, offshore spudcans, pipes, anchors, drilled shafts, driven piles, rock sockets, helical piles, mechanically stabilized earth walls, soil nail walls, slopes and braced excavations) and a wide range of geomaterials from soft clay to soft rock. Two typical limit states (i.e., ULS and SLS) are calibrated. The mean and COV values and number of tests (N) averaged over n-data groups that belong to the same geotechnical structure, limit state, and geomaterial are presented in Figure 4.1.

4.3 Key Observations

- Characterization of ULS model factor received most of the attention in the literature (foundation capacity is the most prevalent), while characterization of SLS model factor is relatively limited. This is because only strength parameters (e.g., cohesion, friction angle, or uniaxial compressive strength) are required in stability analysis that are familiar to engineers and are most often measured in field or laboratory tests.
- 2. Based on the mean of model factor, the bias of calculation model is classified as (1) unconservative (mean<1), (2) moderately conservative (1≤mean≤3), and (3) highly conservative (mean>3). Based on the COV of model factor, the dispersion of calculation model is classified as: (1) low (COV<0.3), (2) medium (0.3≤COV≤0.6), and (3) high (COV>0.6). Note that "low dispersion" means "high precision" and vice-versa. This three-tier classification scheme is deemed reasonable based on the extensive statistical analyses covering numerous geotechnical structures and soil types. It is consistent with the three-tier classification for soil properties

(Phoon et al. 2003), the degree of site and model understanding in the Canadian Highway Bridge Design Code (CSA 2019) and the geotechnical complexity class being considered in the new draft for Eurocode 7 Part 1 - EN 1997-1:202x.



Figure 4.1. Classification of model uncertainty based on the mean (bias) and COV (dispersion) of model factor, where n = number of data groups, N = number of tests averaged over n-data groups, MSE = mechanically stabilized earth wall, MAW = multi-anchor wall, and SNW = soil nail wall. (Image taken from Tang et al. 2020a)

3. Few calculation methods are of low dispersion (or high precision). The model factor COV values for (1) the factor of safety of man-made slopes, (2) the pullout capacity of pipes (laboratory tests on scaled models in 1g condition) and (3) the punch-through capacity of offshore spudcans in stiff-over-soft clays or clay with sand (centrifuge tests that are laboratory tests on scaled models in ng condition) are around 0.3. For these three cases, soil samples are well-prepared, corresponding to lower geotechnical variability. In addition, slope stability is an important and classical problem in geotechnical engineering that has been extensively studied since the 1930s, leading to the better understanding of slope failure mechanism and improved analysis methods (Duncan et al. 2014). For pullout capacity of pipes and punch-through capacity of offshore spudcans, due to the increasing demand of offshore oil and gas, many laboratory tests were performed to study the underlying mechanism and improve the performance of the calculation models as reviewed by Tang and Phoon (2021). Therefore, they

correspond to a high to typical degree of site and model understanding (i.e. low to medium dispersion).

- 4. Calculation methods for the capacity of shallow and deep foundations in soil, steel reinforced earth walls and soil nail walls are of medium dispersion. This is explained by the fact that field load tests or observations are mainly used, corresponding to higher geotechnical variability than laboratory tests. Foundation analysis and design in soil is another important and classical problem in geotechnical engineering that has been studied over one century (e.g., load-displacement response and load-transfer mechanism). They correspond to a typical degree of site and model understanding.
- 5. Calculation methods for foundation settlement, foundation capacity in rock, and geosynthetic reinforced earth walls are of high dispersion (low precision). The reasons may include: (1) soil stiffness is more difficult to predict than strength parameters; (2) only rock compressive strength is incorporated into calculations that is insufficient, as rock mass is usually composed of joints, seams, faults, and bedding planes; and (3) the interaction between geosynthetic and soil is more complicated than that between steel reinforcements and soil. They correspond to a lower degree of site and model understanding.
- 6. Assessment of design methods for large-diameter open-ended piles (LDOEPs) (Petek et al. 2020) and laterally loaded piles (e.g., Phoon and Kulhawy 2005; Briaud and Wang 2018) is relatively limited. This is because significant challenges are involved addressing both problems, such as complicated behaviour (e.g., plug and development of internal friction) and difficulty in LDOEPs installation and obvious deficiencies in current limit state design methods for laterally loaded piles that only cover a specific behavior and lack the ability to properly accommodate both ULS and SLS.

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		Calibra	ation database				Calculation method for	М		
Geo-structure	Limit state	Groun	d condition	Data range	N	Method to determine X _m	X _c /reference	λ	COV	Source
Shallow	Bearing	Clay (natural)	B=0.3-3.1 m	30	Chin (1970)	Hansen (1970)	1.25	0.37	Strahler and
foundation				su=20–139 kPa						Stuedlein (2014)
				B=0.3–5.0 m	42	Hirany and Kulhawy (1988)	Vesić (1973)	1.05	0.29	Tang et al. (2020a)
				D/B=0.0-5.7						
				s _u =9–200 kPa						
		Sand	Controlled	B≤0.1 m	138	Vesić (1963)	AASHTO (2007)	1.67	0.25	Paikowsky et al.
				0.1 <b≤1.0 m<="" td=""><td>21</td><td></td><td></td><td>1.48</td><td>0.39</td><td>(2010)</td></b≤1.0>	21			1.48	0.39	(2010)
			Natural	B>1.0 m	6			1.01	0.23	
				0.1 <b≤1.0 m<="" td=""><td>8</td><td></td><td></td><td>0.99</td><td>0.41</td><td></td></b≤1.0>	8			0.99	0.41	
			Field	B=0.25-4.0 m	113	Housel (1966)	Vesić (1973)	1.33	0.62	Tang et al. (2020a)
			(θ=0°)	D/B=0.0-6.1	106	Hirany and Kulhawy (1988)		1.64	0.47	
					76	10%B		1.77	0.43	
			Chamber	B=0.30–1.22 m	17	Housel (1966)		2.34	0.56	
			(θ=0°)	D/B=0.0-2.0	72	10%B		2.45	0.62	
				φ=27-46°						
			Chamber	B=0.30 m	27	Hirany and Kulhawy (1988)		1.21	0.33	
			(θ>0°)	D/B=0.0-3.0						
				φ=35–40°						
			Centrifuge	B=0.30-7.0 m	48	Peak		1.20	0.35	
			(θ=0°)	D/B=0.0-3.0						
				φ=41-48°						
			Centrifuge	B=0.90–2.54 m	93			1.09	0.22	

Table 4.1. Summary of mean and coefficient of variation (COV) for geotechnical design model factor $M = X_m/X_c$

State-of-the-art review of inherent variability and uncertainty, March 2021 (θ>0°) D/B = 0.0**φ**=41-44° RMR>85 Hirany and Kulhawy (1988) Carter and Kulhawy (1988) 0.84 Paikowsky et Rock (natural) 7 1.81 al. (2010) 65≤RMR<85 22 3.54 0.49 8 44<RMR<65 0.40 11.1 3≤RMR<44 21 24.3 0.55 RMR≥85 7 Goodman (1989) 1.46 0.14 65<RMR<85 22 1.22 0.74 44<u><</u>RMR<65 8 1.06 0.44 $3 \leq RMR < 44$ 21 1.24 0.44 43 Vesić (1963) Effective width Eccentric Sand B=0.05-1.0 m 1.83 0.35 Two-slope 41 1.61 0.40 43 Vesić (1963) 0.42 Full width 1.05 41 Two-slope 0.92 0.46 Vesić (1963) AASHTO (2007) Inclined Sand B=0.05-1.0 m 39 1.43 0.30 Shallow 37 Two-slope AASHTO (2007) 1.29 0.35 Inclined Sand B=0.05-1.0 m Clay (natural) Housel (1966) foundation Tension B=0.38-3.05 m 118 IEEE (2001) 1.15 0.36 Tang et al. (2020a) $s_u = 15 - 300 \text{ kPa}$ Meyerhof and Adams (1968) 74 1.37 0.38 B=0.61-2.5 m 0.33 Sand (natural) 106 IEEE (2001) 1.10 **φ**=30–49° Meyerhof and Adams (1968) 0.42 1.19 Punch-through Sand-over-clay B=0.8-3.0 m 27 Peak 1.49 0.31 and Phoon Load spread (1H:3V) (BSI 2020a) Tang (centrifuge) 2.37 0.38 $H_{s}/B=0.5-3.0$ Load spread (1H:5V) (BSI 2020a) (2019a) $D_r = 88\%, \phi_{cv} = 32^{\circ}$ Punching shear (BSI 2020a) 1.61 0.46 su0=8.7-45 kPa Punching shear (InSafeJIP 2011) 1.69 0.39 Okamura et al. (1998) 0.90 0.12 Ullah et al. (2017) 0.82 0.19

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						J		5,	
	Settlement	Sand (natural)	B=1.0-10.0 m	268	s_m at elastic limit L_1	Terzaghi and Peck (1948)	3.71	0.65	Akbas (2007)
					(Hirany and Kulhawy 1988)	Gibbs and Holtz (1957)	1.37	0.64]
						Alpan (1964)	1.70	1.04	1
						Meyerhof (1965)	2.40	0.90]
						Peck and Bazaraa (1969)	1.62	0.70]
						Peck et al. (1974)	2.76	0.92]
						D'Appolonia et al. (1970)	1.45	0.60]
						Schultze and Sherif (1973)	1.07	0.65]
						Anagnostopoulos et al. (1991)	1.44	0.84]
						Burland and Burbidge (1985)	1.45	0.63	
						Parry (1971)	1.24	0.87	
						Schmertmann et al. (1978)	2.03	0.69	
						Berardi and Lancelotta (1991)	1.55	0.79	
			Bridge	57	All	Schmertmann et al. (1978)	1.21	1.13	Samtani and Allen
				40	All (s _c >12.7 mm)		0.87	0.86	(2018)
				61	All	Hough (1959)	0.90	1.01	
				49	All (s _c >12.7 mm)		0.66	0.45	
Anchor	Pullout	Sand (natural)	B=0.30–2.39 m	45	Housel (1966)	IEEE (2001)	1.48	0.39	Tang et al. (2020a)
						Meyerhof and Adams (1968)	1.45	0.37	
		Sand (controlled)	B=0.10-0.44 m	162		IEEE (2001)	0.94	0.47	
						Meyerhof and Adams (1968)	0.99	0.45	
Pipeline	Upheaval	Sand (1g-model	D _r =5-85%	>300	Peak	Pedersen and Jensen (1988)	0.93	0.21	Stuyts et al. (2016)
	buckling	53%, centrifuge	D/B=0-20			White et al. (2008)	1.05	0.21	
		29%, full-scale 18%)				Byrne et al. (2013)	0.88	0.20	
		Sand (controlled)	B=0.015-0.45 m	143	Peak	Schaminee et al. (1990)	1.21	0.39	Ismail et al. (2018)
			L=0.075-3 m			Bransby et al. (2002)	1.41	0.37	

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			D/B=0.1-8			White et al. (2001)	1.02	0.30	
						DNV (2007)	1.09	0.32	
Offshore	Punch-through	Sand-over-clay	B=3.0-20.0 m	103	Peak	Load spread (1H:3V) (BSI 2020a)	1.89	0.27	Tang and Phoon
spudcan		(centrifuge)	α=0-21°			Load spread (1H:5V) (BSI 2020a)	2.48	0.32	(2019a)
			H _s /B=0.16-1.17			Punching shear (BSI 2020a)	2.86	0.30	
			Dr=44-99%			Punching shear (InSafeJIP 2011)	2.44	0.24	
			$\phi_{cv}=31-34^{\circ}$			Okamura et al. (1998)	0.87	0.22	
			su0=7.2-44.8 kPa			Ullah et al. (2017)	1.02	0.17	
		Clay-sand-clay	B=6.0–16.0 m	28	Peak	Load spread (1H:3V) (BSI 2020a)	1.59	0.18	
		(centrifuge)	α=0-13°			Load spread (1H:5V) (BSI 2020a)	2.02	0.22	
			H _s /B=0.25-1.04			Punching shear (BSI 2020)	1.94	0.23	
			$D_r=44-89\%, \phi_{cv}=31^{\circ}$			Punching shear (InSafeJIP 2011)	1.71	0.17	
			su0=4.4-34 kPa			Okamura et al. (1998)	1.00	0.21	
						Ullah et al. (2017)	1.06	0.13	
Rock socket	Bearing	Rock (natural)	RMR≥85	16	Hirany and Kulhawy (1988)	Carter and Kulhawy (1988)	3.42	0.55	Paikowsky et al.
			65≤RMR<85	35			3.93	0.45	(2010)
			44≤RMR<65	9			6.82	0.92	
			RMR≥85	16		Goodman (1989)	1.59	0.51	
			65≤RMR<85	35			1.40	0.52	
			44≤RMR<65	9			1.47	0.62	
			B=0.10-2.5 m	128	Hirany and Kulhawy (1988)	Teng (1962)	24.4	1.07	Tang et al. (2020a)
			D/B=1.0-31.3			Coates (1967)	1.63	1.07	
			σ _c =0.5–99 MPa			Rowe and Armitage (1987)	1.95	1.07	
			E _m =7.82–75113 MPa			Zhang and Einstein (1998)	1.11	0.86	
			GSI=7.5–95			Asem (2019)	1.78	1.01	
			RQD=20-100%			ARGEMA (1992)	1.23	0.93	

								-	
				118		Abu-Hejleh and Attwooll (2005)	1.74	0.87	
				127		Stark et al. (2017)	1.38	0.81	
				125		Asem et al. (2018)	1.07	0.50	
				265	Maximum load	Teng (1962)	18.4	1.43	
						Coates (1967)	1.23	1.43	
						Rowe and Armitage (1987)	1.47	1.43	
						Zhang and Einstein (1998)	1.02	1.00	
						Asem (2019)	1.38	1.34	
						ARGEMA (1992)	1.40	0.88	
	Shearing		B=0.10-2.44 m	169	Peak	Reynolds and Kaderabek (1980)	1.13	0.70	Tang et al. (2020b)
			D/B=0.54-17.3			Gupton and Logan (1984)	1.70	0.70	
			σ _c =0.48–20 MPa			Rosenberg and Journeaux (1976)	1.23	0.79	
			E _m =47.6–6061 MPa			Horvath and Kenney (1979)	2.20	0.80	
						Williams et al. (1980)	1.18	0.89	
						Rowe and Armitage (1987)	1.01	0.80	
						Kulhawy and Phoon (1993)	1.01	0.80	
						Miller (2003)	1.14	0.80	
						AASHTO (2017)	1.43	0.80	
						Asem and Gardoni (2019b)	1.19	0.60	
						Xu et al. (2020)	1.39	0.88	
						Meigh and Wolski (1979)	1.97	0.73	
						Abu-Hejleh and Attwooll (2005)	1.31	0.70	
	Settlement	Rock (natural)		37	s _m at Q=50%Q _{L2}	Kulhawy (1978)	1.64	1.73	Muganga (2008)
Steel H pile	Bearing	Clay (natural)	B=0.28-0.41 m	26	Davisson (1972)	API (1974)	1.26	0.56	Tang and Phoon
			D/B=16-95			Burland (1973)	0.96	0.61	(2018a)
				16		Vijayvergiya and Focht (1972)	0.74	0.39	Paikowsky et al.

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				17		Tomlinson (1986)	0.82	0.40	(2004)
				16		API (1989)	0.90	0.41	
		Sand (natural)	B=0.28-0.42 m	36	Davisson (1972)	SPT-Meyerhof (1976)	1.52	0.66	Tang and Phoon
			D/B=22-110			Burland (1973)	0.78	0.47	(2018a)
						Nordlund (1963)	0.82	0.52	
				19		Nordlund (1963)	0.94	0.40	Paikowsky et al.
				18		SPT-Meyerhof (1976)	0.81	0.38	(2004)
				19		Esrig and Kirby (1979)	0.78	0.51	
				18		SPT-97	1.35	0.43	
		Mixed (natural)	B=0.28-0.42 m	29		Burland (1973)	0.81	0.40	Tang and Phoon
			D/B=17-85						(2018a)
				20		Tomlinson (1986)/Nordlund	0.59	0.39	Paikowsky et al.
						(1963)			(2004)
				34		API (1989) /Nordlund (1963)	0.79	0.44	
				32		Esrig and Kirby (1979)	0.48	0.48	
				40		SPT-97	1.23	0.45	
Steel pipe pile	Bearing	Clay (natural)	B=0.1-0.81 m	110	10%B	BSI (2020b)	1.02	0.32	Tang and Phoon
			D/B=7.9-200			NGI-05 (Karlsrud et al. 2005)	1.10	0.29	(2019b)
			PI=11-160%			SHANSEP (Saye et al. 2013)	1.14	0.27	
			OCR=1-43.2			ICP-05 (Jardine et al. 2005)	1.06	0.28	
			S _t =1-17						
				18	Davisson (1972)	Tomlinson (1986)	0.64	0.50	Paikowsky et al.
				19		API (1989)	0.79	0.54	(2004)
				12		Esrig and Kirby (1979)	0.45	0.60	
				19		Vijayvergiya and Focht (1972)	0.67	0.55	
				12		SPT-97	0.39	0.62	

		Sand (natural)	B=0.14-0.76 m	68	10%B	BSI (2020b)	1.11	0.54	Tang and Phoon
			D/B=13-251			NGI-05 (Clausen et al. 2005)	1.05	0.41	(2018b, 2019b)
			φ=30-42°	29		ICP-05 (Jardine et al. 2005)	1.13	0.30	
			D _r =15-93%			Fugro-05 (Kolk et al. 2005)	0.95	0.36	
						UWA-05 (Lehane et al. 2005)	1.08	0.37	
				19	Davisson (1972)	Nordlund (1963)	1.48	0.52	Paikowsky et al.
				20		Esrig and Kirby (1979)	1.18	0.62	(2004)
				20		SPT-Meyerhof (1976)	0.94	0.59	
				19		SPT-97	1.58	0.52	
		Mixed (natural)		13		Tomlinson (1986)/Nordlund	0.74	0.59	
						(1963)			
				32		API (1989)/Nordlund (1963)	0.80	0.45	
				29		Esrig and Kirby (1979)	0.54	0.48	
				33		SPT-97	0.76	0.38	
Concrete pile	Bearing	Clay (natural)	B=0.1-0.81 m	65	10%B	BSI (2020b)	1.09	0.34	Tang and Phoon
			D/B=7.9-200			NGI-05 (Karlsrud et al. 2005)	0.95	0.26	(2019b)
			PI=11-160%			SHANSEP (Saye et al. 2013)	1.01	0.34	
			OCR=1-43.2			ICP-05 (Jardine et al. 2005)	1.04	0.35	
			St=1-17						
				18	Davisson (1972)	Vijayvergiya and Focht (1972)	0.76	0.29	Paikowsky et al.
				17		API (1989)	0.81	0.26	(2004)
				8		Esrig and Kirby (1979)	0.81	0.51	
				18		Tomlinson (1986)	0.87	0.48	
		Sand (natural)	B=0.14-0.76 m	50	10%B	BSI (2020b)	0.95	0.37	Tang and Phoon
			D/B=13-251			NGI-05 (Clausen et al. 2005)	0.83	0.33	(2018b, 2019b)
			φ=30-42°	40]	ICP-05 (Jardine et al. 2005)	1.13	0.29	

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		D _t =15-93%			Fugro-05 (Kolk et al. 2005)	0.87	0.41		
					UWA-05 (Lehane et al. 2005)	1.00	0.33		
			36	Davisson (1972)	Nordlund (1963)	1.02	0.48	Paikowsky et	al.
			35		Esrig and Kirby (1979)	1.10	0.44	(2004)	
			36		SPT-Meyerhof (1976)	0.61	0.61		
			36		SPT-97	1.21	0.47	1	
	Mixed (natural)		33		Tomlinson (1986)/Nordlund	0.96	0.49	1	
					(1963)				
			80	-	API (1989)/Nordlund (1963)	0.87	0.48]	
			80		Esrig and Kirby (1979)	0.81	0.38	1	
			71		SPT-97	1.81	0.50	1	
			30		Nottingham and Schmertmann	0.84	0.31	1	
					(1975)				
		B=0.356-0.914 m	80	Modified Davisson	Bustamante and Gianeselli (1982)	1.07	0.39	Amirmojahedi	and
		D=11-61 m		(AASHTO 2017)	Nottingham and Schmertmann	1.21	0.35	Abu-Farsakh (20)19)
					(1975)				
					De Ruiter and Beringen (1979)	0.95	0.36		
					Price and Wardle (1982)	0.83	0.34		
					Fugro-05 (Kolk et al. 2005;	1.34	0.45		
					Van Dijk and Kolk 2010)				
					ICP-05 (Jardine et al. 2005)	1.33	0.45		
					NGI-05 (Clausen et al. 2005;	1.24	0.45		
					Karlsrud et al. 2005)				
					Tumay and Fakhroo (1982)	1.36	0.35		
					Aoki and Velloso (1975)	0.77	0.51		
					Salgado et al. (2011)	1.29	0.56		

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						UWA-05 (Lehane et al. 2005,	1.17	0.31	
						2013)			
Concrete/steel	Bearing	Sand (natural)	B=0.235–1.372 m	43	Hansen (1963)-80%	CPT-Meyerhof (1983)	0.91	0.45	Moshfeghi and
pile			D/B=17.1-85.2			Nottingham and Schmertmann	0.86	0.31	Eslami (2018)
						(1975)			
						De Ruiter and Beringen (1979)	0.69	0.41	
						Kempfert and Becker (2010)	0.98	0.41	
						Bustamante and Gianeselli (1982)	0.93	0.43	
						Eslami and Fellenius (1997)	1.12	0.19	
						Fugro-05 (Kolk et al. 2005)	1.00	0.43	
						ICP-05 (Jardine et al. 2005)	1.00	0.43	
						NGI-05 (Clausen et al. 2005)	1.12	0.42	
						UWA-05 (Lehane et al. 2005)	0.88	0.40	
	Bearing/	Mixed (natural)	B=0.114–1.50 m	60	Hansen (1963)-80%	Schmertmann (1978)	1.12	0.35	Heidarie Golafzani et
	Tension		D/B=10.3-111			De Ruiter and Beringen (1979)	1.14	0.41	al. (2020)
								0.47	
						Bustamante and Gianeselli (1982)	1.29	0.47	
						CPT-Meyerhof (1983)	1.29 0.99	0.47	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)	1.29 0.99 0.99	0.47 0.44 0.33	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)	1.29 0.99 0.99 1.13	0.47 0.44 0.33 0.52	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)	1.29 0.99 0.99 1.13 0.98	0.47 0.44 0.33 0.52 0.43	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)Bazaraa and Kurkur (1986)	1.29 0.99 0.99 1.13 0.98 1.57	0.47 0.44 0.33 0.52 0.43 0.50	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)Bazaraa and Kurkur (1986)Briaud and Tucker (1988)	1.29 0.99 0.99 1.13 0.98 1.57 0.85	0.47 0.44 0.33 0.52 0.43 0.50 0.40	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)Bazaraa and Kurkur (1986)Briaud and Tucker (1988)Décourt (1995)	1.29 0.99 0.99 1.13 0.98 1.57 0.85 1.12	0.47 0.44 0.33 0.52 0.43 0.50 0.40 0.54	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)Bazaraa and Kurkur (1986)Briaud and Tucker (1988)Décourt (1995)API (2000)	1.29 0.99 0.99 1.13 0.98 1.57 0.85 1.12 1.14	0.47 0.44 0.33 0.52 0.43 0.50 0.40 0.54 0.44	
						Bustamante and Gianeselli (1982)CPT-Meyerhof (1983)Eslami and Fellenius (1998)SPT-Meyerhof (1976)Shioi and Fukui (1982)Bazaraa and Kurkur (1986)Briaud and Tucker (1988)Décourt (1995)API (2000)CGS (2006)	1.29 0.99 0.99 1.13 0.98 1.57 0.85 1.12 1.14 0.87	0.47 0.44 0.33 0.52 0.43 0.50 0.40 0.54 0.40 0.54 0.44	

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		Sand (natural)	D=3.05–55 m	114		Nordlund (1963)	1.60	0.88	(2018)
		Mixed (natural)		58		Tomlinson (1986)/Nordlund	1.43	0.94	
						(1963)			
RC		Soil (natural)		29		Tomlinson (1986)/Nordlund	2.30	0.70	
						(1963)			
SC				135		Tomlinson (1986)/Nordlund	1.28	0.65	
						(1963)			
Steel H pile				9		Tomlinson (1986)/Nordlund	0.90	0.56	
						(1963)			
SPC				27		Tomlinson (1986)/Nordlund	1.45	0.96	
						(1963)			
SPO				11		Tomlinson (1986)/Nordlund	2.37	1.24	
						(1963)			
Helical pile	Bearing	Clay (natural)	d=38–114 mm	176	10%B+QL/AE	Capacity-to-torque correlation	1.10	0.23	Tang and Phoon
		Sand (natural)	D/B=3-70	115			1.39	0.33	(2018c)
		Clay (natural)	d=38–57 mm	53		Individual plate bearing	1.25	0.41	
		Sand (natural)	D/B=8.4-110	49			1.46	0.42	
		Clay/sand (natural)	d=38-89 mm	27		Least value of individual plate	1.79	0.50	Cherry and Souissi
				27		bearing, cylindrical shear and	1.79	0.60	(2010)
				21		capacity-to-torque correlation	1.78	0.54	
				18			1.99	0.52	
		Clay/sand (natural)	d=0.168-0.508 m	83	5%B	Capacity-to-torque correlation	1.25	0.36	Tang and Phoon
			B=0.356-1.016 m	47		CGS (2006)	1.17	0.36	(2018c, 2020)
			D/B=4.1-20	47		ISHF (Lutenegger 2015)	1.06	0.45	
			s ₀ =12-305 kPa	47		BSI (2016)	1.04	0.35	
			~u	.,		D D1 (2010)	1.01	0.55	

			S/B=1.5-4.5						
		Soil/rock (natural)		46	NA	Bearing (SPT) and cylindrical shear	1.06	0.55	Perko (2009)
Drilled shaft	Bearing	Clay (natural)	B=0.35-1.52 m D/B= 1.6-56 s _u =41-246 kPa	64	Modified Davisson (AASHTO 2017)	Brown et al. (2010)	1.41	0.63	Tang et al. (2019)
			B≥0.6 m	22	Davisson (1972)		1.02	0.41	AbdelSalam et al. (2015)
				53		Reese and O'Neill (1988)	0.90	0.47	Paikowsky et al.
				13			0.84	0.50	(2004)
				40			0.88	0.48	
		Sand (natural)	B=0.35-2.00 m D/B=5.1-59 φ=30-41°	44	Modified Davisson (AASHTO 2017)	Brown et al. (2010)	1.19	0.39	Tang et al. (2019)
			B≥0.6 m	45	Davisson (1972)		0.91	0.40	AbdelSalam et al. (2015)
			B=0.458-2.135 m	24	5%B	O'Neill and Reese (1999)	1.14	0.58	Ng and Fazia (2012)
			D/B=4.05-45.3			Brown et al. (2010)	1.21	0.60	
			B=1.0-1.8 m	11	Davisson (1972)	O'Neill and Reese (1999)	0.60	0.58	Zhang and Chu
			D/B=11.2-64.2	17			1.06	0.28	(2009a)
				32		Reese and Wright (1977)	1.22	0.67	Paikowsky et al.
				12			1.45	0.50	(2004)
				9			1.32	0.62	
				32		Reese and O'Neill (1988)	1.71	0.60	
				12			2.27	0.46	
				9			1.62	0.74	

		Mixed (natural)	B≥0.6 m		90	Davisson (1972)	Brown et al. (2010)	0.81	0.37	AbdelSalam et al.
										(2015)
			B=0.61-1.83 m		34	5%B	O'Neill and Reese (1999)	1.27	0.30	Abu-Farsakh et al.
			D/B=7.2-34.5				Brown et al. (2010)	0.99	0.30	(2013)
					44	Davisson (1972)	Reese and O'Neill (1988)	1.19	0.30	Paikowsky et al.
					21			1.04	0.29	(2004)
					12			1.32	0.28	
					10			1.29	0.27	
					44		Reese and Wright (1977)	1.09	0.35	
					21			1.01	0.42	
					12			1.20	0.32	
					10			1.16	0.25	
		Gravel (natural)	B=0.59-1.50 m		41	Modified Davisson	Brown et al. (2010)	1.69	0.47	Tang et al. (2019)
			D/B=6.2-30			(AASHTO 2017)				
			φ=37-47°							
Pile	Bearing	Soil (natural)	B=0.36-0.46	m	68	10%B+QL/AE	Bustamante and Gianeselli (1983)	1.15	0.43	Briaud and Tucker
foundation			(square concrete)		15		Bustamante and Gianeselli (1982)	1.32	0.44	(1988)
			B=0.30-0.41	m	77		Coyle and Castello (1981)	1.19	0.66	
			(drilled shaft)		63		MSHD (1972)	1.15	0.70	
			D=3.0–25.0 m		53		Briaud and Tucker (1984)	1.40	0.51	
					68		De Ruiter and Beringen (1979)	1.49	0.42	
					68		Clisby et al. (1978)	0.72	0.38	
					77		API (1984)	0.92	0.58	
					68		Schmertmann (1978)	1.48	0.74	
					23		Tumay and Fakhroo (1982)	1.99	0.43	
					53		SPT-Meyerhof (1976)	1.73	0.72	

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				68		Briaud et al. (1986)	2.78	0.59	
Steel pipe pile	Tension	Clay (natural)	B=0.1-0.81 m	64	10%B	BSI (2020b)	0.88	0.28	Tang and Phoon
			D/B=12-110			NGI-05 (Karlsrud et al. 2005)	0.99	0.26	(2019b)
			PI=12-110%			SHANSEP (Saye et al. 2013)	1.04	0.30	
			OCR=1-43.2			ICP-05 (Jardine et al. 2005)	1.02	0.34	
			$S_t = 1 - 8.3$						
		Sand (natural)	B=0.25-0.76 m	63		BSI (2020b)	1.15	0.61	
			D/B=19-84			NGI-05 (Clausen et al. 2005)	1.16	0.49	
			φ=30-42°	40		ICP-05 (Jardine et al. 2005)	1.26	0.36	Tang and Phoon
			D _r =31-97%		Fugro-05 (Kolk et al. 2005)	Fugro-05 (Kolk et al. 2005)	1.49	0.77	(2018b)
						UWA-05 (Lehane et al. 2005)	1.20	0.36	
Drilled shaft	Tension	Clay (natural)	B=0.36–1.80 m	32	Modified Davisson	Brown et al. (2010)	1.11	0.28	Tang et al. (2019)
			D/B=3.4-55		(AASHTO 2017)				
			s _u =21-250 kPa						
				13	Davisson (1972)	Reese and O'Neill (1988)	0.87	0.37	Paikowsky et al.
									(2004)
		Sand (natural)	B=0.30-1.31 m	30	Modified Davisson	Brown et al. (2010)	1.28	0.33	Tang et al. (2019)
			D/B=2.5-43		(AASHTO 2017)				
			φ=30-45°						
				11	Davisson (1972)	Reese and O'Neill (1988)	1.09	0.51	Paikowsky et al.
						Reese and Wright (1977)	0.83	0.54	(2004)
		Gravel	B=0.43-2.26 m	109	Modified Davisson	Brown et al. (2010)	1.14	0.43	Tang et al. (2019)
			D/B=1.77-17.3		(AASHTO 2017)				
			φ=42-48°						
				14	Davisson (1972)	Reese and O'Neill (1988)	1.25	0.29	Paikowsky et al.
						Reese and Wright (1977)	1.24	0.41	(2004)

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				16		Carter and Kulhawy (1988)	1.18	0.46	
						O'Neill and Reese (1999)	1.25	0.37	
				39		Reese and O'Neill (1988)	1.08	0.41	
				25		Reese and Wright (1977)	1.07	0.48	
Helical pile	Tension	Clay (natural)	d=38–114 mm	147	10%B+QL/AE	Capacity-to-torque correlation	0.95	0.27	Tang and Phoon
		Sand (natural)	D/B=10-62	105			1.09	0.31	(2018c)
		Clay/sand (natural)	d=38-89 mm	91	NA	Cylindrical shear	1.50	0.79	Hoyt and Clemence
			B=0.152-0.508 m			Individual plate bearing	1.56	0.82	(1989)
			n=2-14			Capacity-to-torque correlation	1.49	0.59	
			d=38-89 mm	25	10%B+QL/AE	Least value of individual plate	1.43	0.43	Cherry and Souissi
				39		bearing, cylindrical shear and	1.31	0.54	(2010)
				20		capacity-to-torque correlation	1.56	0.43	
				25			2.08	0.53	
		Clay/sand (natural)	d=0.168-0.406 m	28	5%B	Capacity-to-torque correlation	0.92	0.38	Tang and Phoon
			B=0.304-1.016 m	31		CGS (2006)	1.26	0.32	(2018c, 2020)
			D/B=4.2-24	31		ISHF (Lutenegger 2015)	1.22	0.39	
			su=24-300 kPa	31		BSI (2020b)	1.18	0.33	
			φ=30–45°, n=2–6						
			S/B=1.5-3						
		Soil/rock (natural)		66	NA	Bearing (SPT) and cylindrical	0.87	0.46	Perko (2009)
						shear			
		Clay/sand (natural)	d=0.089-0.406 m	36	Average of failure loads	Meyerhof (1983)	1.79	0.71	Fateh et al. (2017)
			B=0.355-0.762 m		interpreted by Modified	Schmertmann (1978)	1.42	0.49	
			D/B=6.43-23.9		Davisson (AASHTO 2017),	De Ruiter and Beringen (1979)	1.4	0.54	
			n=1-5		Hansen (1963)-80%, and	Bustamante and Gianeselli (1982)	1.06	0.51	
			S/B=1.5-4.5		Mazurkiewicz (1972)	Eslami and Fellenius (1997)	1.98	0.55	

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Fugro-05 (Kolk et al. 2005; Van 1.58 0.52 methods Dijk and Kolk 2010) ICP-05 (Jardine et al. 2005) 0.84 0.68 UWA-05 (Lehane et al. 2005, 0.9 0.46 2013) NGI-05 (Clausen et al. 2005; 1.07 0.39 Karlsrud et al. 2005) Kempfert and Becker (2010) 1.71 0.56 Clay/sand d=0.114-0.406 m Tappenden (2007) 23 Mazurkiewicz (1972) Bustamante and Gianeselli (1982) 0.76 0.46 (natural) B=0.356-0.914 m D/B=6.43-18.8 n=1-3, S/B=1.5-3 Clay (natural) 47 NA Individual plate bearing 1.03 Perko (2009) d=73–114 mm 0.46 B=0.203-0.457 m 54 Sand (natural) 1.16 0.72 D/B=2.57-66 54 Individual plate bearing (SPT) 1.34 0.61 Clay (natural) n = 1 - 432 Cylindrical shear 0.82 0.32 Sand (natural) 42 Cylindrical shear 1.07 0.54 47 Lest value of individual plate Clay (natural) 1.03 0.46 Sand (natural) Bearing (SPT) and cylindrical 54 1.34 0.6 shear Soil/rock (natural) 112 Bearing (SPT) and cylindrical 0.97 0.53 shear Drilled shaft Clay (controlled) B=0.089-0.175 m 45 Reese (1958) 0.90 Phoon and Kulhawy Lateral Lateral or moment limit 0.24 D/B=3.00-7.98 (Hirany and Kulhawy 1988) 0.25 (2005)Hansen (1961) 1.22 h/D=0.03-4.01 Broms (1964a) 1.46 0.36 Stevens and Audibert (1979) 0.70 0.24

				Dandalah and Haulaha (1094)	0.94	0.04	
				Randolph and Houisby (1984)	0.84	0.24	1
Clay (natural)	B=0.08-1.98 m	27	-	Reese (1958)	0.94	0.35	
	D/B=2.25-10.49			Hansen (1961)	1.24	0.32	
	h/D=0.03-6.83			Broms (1964a)	1.55	0.42	
				Stevens and Audibert (1979)	0.73	0.33	
				Randolph and Houlsby (1984)	0.87	0.34	
Clay (controlled)	B=0.089-0.175 m	47	Chin (1970)	Reese (1958)	1.43	0.26	
	D/B=3.00-7.98			Hansen (1961)	1.95	0.28	
	h/D=0.03-4.01			Broms (1964a)	2.28	0.35	
				Stevens and Audibert (1979)	1.12	0.28	
				Randolph and Houlsby (1984)	1.33	0.27	
Clay (natural)	B=0.08-1.98 m	27		Reese (1958)	1.40	0.33	
	D/B=2.25-10.49			Hansen (1961)	1.85	0.31	
	h/D=0.03-6.83			Broms (1964a)	2.29	0.41	
				Stevens and Audibert (1979)	1.09	0.32	
				Randolph and Houlsby (1984)	1.30	0.32	
Sand (controlled)	B=0.076-0.152 m	53	Lateral or moment limit	Hansen (1961)	0.71	0.36	
	D/B=2.61-9.03		(Hirany and Kulhawy 1988)	Broms (1964b)	1.20	0.42	
	h/D=0.06-4.99			Reese et al. (1974)	0.82	0.51	
Sand (natural)	B=0.05-1.62 m	22		Hansen (1961)	0.56	0.39	
	D/B=2.49-7.03			Broms (1964b)	1.26	0.35	
	h/D=0.00-5.37			Reese et al. (1974)	0.81	0.39	
Sand (controlled)	B=0.076-0.152 m	55	Chin (1970)	Hansen (1961)	1.05	0.32	I
	D/B=2.61-9.03			Broms (1964b)	1.77	0.44	I
	h/D=0.06-4.99			Reese et al. (1974)	1.19	0.48	1
Sand (natural)	B=0.05-1.62 m	22		Hansen (1961)	0.83	0.30	
	Clay (natural) Clay (controlled) Clay (natural) Clay (natural) Sand (controlled) Sand (controlled) Sand (natural) Sand (natural)	Clay (natural) $B=0.08-1.98 \text{ m}$ D/B=2.25-10.49 h/D=0.03-6.83 Clay (controlled) $B=0.089-0.175 \text{ m}$ D/B=3.00-7.98 h/D=0.03-4.01 Clay (natural) $B=0.08-1.98 \text{ m}$ D/B=2.25-10.49 h/D=0.03-4.01 Clay (natural) $B=0.08-1.98 \text{ m}$ D/B=2.25-10.49 h/D=0.03-6.83 Sand (controlled) $B=0.076-0.152 \text{ m}$ D/B=2.61-9.03 h/D=0.06-4.99 Sand (natural) $B=0.05-1.62 \text{ m}$ D/B=2.49-7.03 h/D=0.00-5.37 Sand (controlled) $B=0.076-0.152 \text{ m}$ D/B=2.61-9.03 h/D=0.00-5.37 Sand (controlled) $B=0.076-0.152 \text{ m}$ D/B=2.61-9.03 h/D=0.06-4.99 Sand (controlled) $B=0.076-0.152 \text{ m}$ D/B=2.61-9.03 h/D=0.06-4.99 Sand (controlled) $B=0.076-0.152 \text{ m}$ D/B=2.61-9.03 h/D=0.06-4.99 Sand (natural) $B=0.05-1.62 \text{ m}$ D/B=2.61-9.03 h/D=0.06-4.99	Clay (natural) $B=0.08-1.98 \text{ m}$ $D/B=2.25-10.49$ $h/D=0.03-6.83$ 27Clay (controlled) $B=0.089-0.175 \text{ m}$ $D/B=3.00-7.98$ $h/D=0.03-4.01$ 47Clay (natural) $B=0.08-1.98 \text{ m}$ $D/B=2.25-10.49$ $h/D=0.03-6.83$ 27Sand (controlled) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.06-4.99$ 53Sand (natural) $B=0.05-1.62 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.00-5.37$ 22Sand (controlled) $B=0.076-0.152 \text{ m}$ $D/B=2.49-7.03$ $h/D=0.00-5.37$ 55Sand (controlled) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.00-4.99$ 55Sand (controlled) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.06-4.99$ 55Sand (natural) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.06-4.99$ 55Sand (natural) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.06-4.99$ 55Sand (natural) $B=0.076-0.152 \text{ m}$ $D/B=2.61-9.03$ $h/D=0.06-4.99$ 55	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 D/B=2.25-10.49 h/D=0.03-6.83 Rese (1958) Clay (controlled) B=0.089-0.175 m D/B=3.00-7.98 h/D=0.03-4.01 47 D/B=3.00-7.98 h/D=0.03-4.01 Chin (1970) Rese (1958) Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 D/B=2.25-10.49 h/D=0.03-6.83 Chin (1970) Rese (1958) Clay (natural) B=0.076-0.152 m D/B=2.25-10.49 27 D/B=2.25-10.49 Rese (1958) Hansen (1961) Broms (1964a) Stevens and Audibert (1979) Randolph and Houlsby (1984) Rese (1958) Sand (controlled) B=0.076-0.152 m D/B=2.61-9.03 h/D=0.06-4.99 53 D/B=2.61-9.03 h/D=0.00-5.37 Lateral or moment limit (Hirany and Kulhawy 1988) Broms (1964b) Sand (natural) B=0.076-0.152 m D/B=2.49-7.03 h/D=0.00-5.37 55 D/B=2.49-7.03 h/D=0.06-4.99 Chin (1970) Hansen (1961) Sand (controlled) B=0.076-0.152 m D/B=2.61-9.03 h/D=0.06-4.99 55 D/B=2.61-9.03 h/D=0.06-4.99 Chin (1970) Hansen (1961) Sand (natural) B=0.05-1.62 m D/B=2.61-9.03 h/D=0.06-4.99 55 D/B=2.61-9.03 h/D=0.06-4.99 Chin (1970) Hansen (1961) Sand (natural) B=0.05-1.62 m D/B=2.61-9.03 h/D=0.06-4.99 55 D/B=2.61-9.03 h/	Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 D/B=2.25-10.49 h/D=0.03-6.83 Recse (1958) 0.94 Hansen (1961) 1.24 Broms (1964a) 1.55 Stevens and Audibert (1979) 0.73 Randolph and Houlsby (1984) 0.87 Clay (controlled) B=0.089-0.175 m D/B=3.00-7.98 h/D=0.03-4.01 47 D/B=3.00-7.98 h/D=0.03-4.01 Chin (1970) Recse (1958) 1.43 Hansen (1961) 1.95 Broms (1964a) 2.28 Stevens and Audibert (1979) 1.12 Randolph and Houlsby (1984) 1.33 Recse (1958) Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 D/B=2.25-10.49 h/D=0.03-6.83 Afficial for the form of the form (1964a) 2.29 Stevens and Audibert (1979) 1.12 Randolph and Houlsby (1984) 1.30 Sand (controlled) B=0.076-0.152 m D/B=2.05-1.62 m 53 D/B=2.49-7.03 h/D=0.06-4.99 Lateral or moment limit (Hirany and Kulhawy 1988) Hansen (1961) 0.71 Broms (1964b) 1.20 Rees et al. (1974) 0.82 Broms (1964b) 1.20 Rees et al. (1974) 0.82 Broms (1964b) 1.20 Rees et al. (1974) 0.81 Broms (1964b) 1.20 Rees et al. (1974) 0.81 Broms (1964b) 1.27 Rees et al. (1974) 0.81 Broms (1964b) 1.27 Rees et al. (1974) 0.81 Broms (1964b) 1.27 Rees et al. (1974) 0.81 Broms (1964b) 1.27 Sand (controlled)<	Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 D/B=2.25-10.49 h/D=0.03-6.83 Resc (1958) 0.94 0.35 Hansen (1961) 1.24 0.32 Broms (1964a) 1.55 0.42 Stevens and Audibert (1979) 0.73 0.33 Randolph and Houlsby (1984) 0.87 0.34 Clay (controlled) B=0.089-0.175 m D/B=3.00-7.98 h/D=0.03-4.01 47 D/B=3.00-7.98 h/D=0.03-4.01 Chin (1970) Reese (1958) 1.43 0.26 Hansen (1961) 1.95 0.28 Broms (1964a) 2.28 0.35 Stevens and Audibert (1979) 1.12 0.28 Randolph and Houlsby (1984) 1.33 0.27 Clay (natural) B=0.08-1.98 m D/B=2.25-10.49 h/D=0.03-6.83 27 No Resc (1958) 1.40 0.33 Sand (controlled) B=0.076-0.152 m D/B=2.61-9.03 53 Lateral or moment limit (Hirany and Kulhawy 1988) Hansen (1961) 0.71 0.36 Sand (natural) B=0.05-1.62 m D/B=2.49-7.03 h/D=0.06-4.99 22 Marsen (1961) 0.82 0.51 Sand (controlled) B=0.05-1.62 m D/B=2.61-9.03

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			D/B=2.49-7.03			Broms (1964b)	1.89	0.33	
			h/D=0.00-5.37			Reese et al. (1974)	1.19	0.30	
Driven pile	Settlement	Clay (natural)	su=9.6–500 kPa	21	s _m at Q=50%Q _{uc}	Coyle (Briaud et al. 1986)	1.26	0.45	Briaud and Tucker
				12		Penpile (Clisby et al. 1978)	1.51	0.35	(1988)
				30		Verbrugge (1981)	1.82	0.44	
Pile				22		'FZ' model (Frank and Zhao 1982)	1.41	0.44	Abchir et al. (2016)
foundation						'AB1' model (Abchir et al. 2016)	0.78	0.40	
						'AB2' model (Abchir et al. 2016)	0.66	0.67	
Driven pile		Mixed (natural)		23	s _m at Q=50%Q _{uc}	Coyle (Briaud et al. 1986)	0.44	1.30	Briaud and Tucker
				10		Penpile (Clisby et al. 1978)	0.99	0.72	(1988)
				22		Briaud and Tucker (1988)	0.57	1.07	
				19		Verbrugge (1981)	0.71	0.77	
				33		Coyle (Briaud et al. 1986)	1.49	0.76	
				20		Penpile (Clisby et al. 1978)	1.56	0.60	
				32		Briaud and Tucker (1988)	1.18	1.01	
				27		Verbrugge (1981)	1.02	0.63	
CEP				29	s _m at Q=50%Q _{DA}	Load transfer analysis	1.18	0.78	Gurbuz (2007)
						Elastic solution (Poulos 1994)	1.11	0.65	
CFA				31		Load transfer analysis	0.94	0.50	
Pile				62		'FZ' model (Frank and Zhao 1982)	1.23	0.64	Abchir et al. (2016)
foundation						'AB1' model (Abchir et al. 2016)	1.02	0.71	
						'AB2' model (Abchir et al. 2016)	0.88	0.99	
Drilled shaft		Soil (natural)	B=1.0–1.8 m	24	s_m at Q=50% Q _{DA}	Vesić (1977)	0.24	0.38	Zhang and Chu
			D/B=11.2-64.2	12		Mayne and Harris (1993)	0.64	0.22	(2009b)
				19		Reese and O'Neill (1989)	1.80	0.31	
				19		Load transfer curves using Hong	0.90	0.39	

						-		-	
						Kong beta method			
				12		Load transfer curves using	1.05	0.41	
						correlations with SPT blow count			
		Rock (natural)	B=1.0–1.5 m	14		Vesić (1977)	0.87	0.30	
			D/B=17.5-58.3	14		Kulhawy and Carter (1992)	0.81	0.24	
			σc=15-202 MPa			(100%)			
			RQD=29-100%	14		Kulhawy and Carter (1992) (85%)	1.01	0.24	
			RMR=17-79%	14		Kulhawy and Carter (1992) (70%)	1.22	0.34	
				14		Load transfer method using the	1.21	0.30	
						ASTM correlation			
				5		Load transfer method using	0.57	0.61	
						correlation with RMR			
				14		Load transfer method using	1.11	0.30	
						correlation with RQD			
Displacement		Soil (natural)	B=0.156–1.02 m	51	s _m at Q=40%Q _{uc}	'FZ' model (Frank and Zhao 1982)	1.26	0.56	Abchir et al. (2016)
pile			D=4.0-54.0 m			'AB1' model (Abchir et al. 2016)	0.93	0.60	
						'AB2' model (Abchir et al. 2016)	0.81	0.77	
Replacement			B=0.196–1.92 m	39	s _m at Q=40%Q _{uc}	'FZ' model (Frank and Zhao 1982)	1.26	0.66	Abchir et al. (2016)
pile			D=3.5-43.4 m			'AB1' model (Abchir et al. 2016)	1.02	0.75	
						'AB2' model (Abchir et al. 2016)	0.89	1.09	
Pile			B=0.156–1.92 m	90		'FZ' model (Frank and Zhao 1982)	1.26	0.60	
foundation			D=3.5–54.0 m			'AB1' model (Abchir et al. 2016)	0.97	0.66	
						'AB2' model (Abchir et al. 2016)	0.84	0.90	
Slope	Global	Real case histories	θ=2-90°	83	FS _A =1	Direct	1.07	0.21	Travis et al. (2011)
	stability		LL=19-197%	134	1	Bishop (1955)	1.00	0.20	
			PL=5-107%	43	1	Force	0.95	0.20	

			PI=6-105%	41		Complete	0.97	0.15	
		Embankment (fill)	θ=18-47°	27	FS _A =1	Bishop (1955)	1.11	0.28	Bahsan et al. (2014)
			H=2–18.7 m			Spencer (1967)	1.19	0.27	
		Excavation (cut)	θ=27-45°	7		Bishop (1955)	0.89	0.28	
			H=5–18.8 m			Spencer (1967)	0.90	0.26	
		Natural slope	θ=26-57°	9		Bishop (1955)	1.41	1.00	
			H=7.6–34 m			Spencer (1967)	1.57	0.96	
Excavation	Base heave	Real case histories	H _e =3.0–19.7 m	24	FS _A =0.8	Modified Terzaghi (1943)	1.02	0.16	Wu et al. (2014)
	stability		H _d =0.3-23.7 m		(total failure)	Bjerrum and Eide (1956)	1.09	0.15	
			Be=4.0-70.0 m		FS _A =1.2	Slip circle (JSA 1988)	1.27	0.22	
					(near failure)				
					FS _A =1.2				
					(non-failure)				
MSE	Load	Geosynthetic	Cohesionless backfill	114		AASHTO (2014)	0.45	0.92	Allen and Bathurst
			Cohesive backfill	79			0.16	1.46	(2018)
			All soil	193			0.33	1.14	
			Stiff wall face (sand)	73			0.33	0.96	
			Flexible wall face	41			0.66	0.73	
			(sand)						
			Battered wall (all	50			0.58	0.92	Allen and Bathurst
			soil)						(2015)
			Vertical wall (all	143			0.24	1.05	
			soil)						
		Steel strip	Strip (\$>45°)	21		PWRC (2003)	2.57	0.44	Miyata and Bathurst
			Strip (35<∳≤45°)	93			1.12	0.33	(2012a)
			Strip (∳≤35°)	40			0.53	0.48	

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		Strip	104	AASHTO (2014)	1.29	0.58	Allen and Bathurst
		Bar mat	29		0.85	0.41	(2018)
		Welded wire	52		1.07	0.4	
		All	185		1.16	0.55	
		Cohesionless soil	104	Allen and Bathurst (2015, 2018)	0.95	0.31	Bozorgzadeh et al.
				Bathurst et al. (2013)	1.00	0.47	(2020)
				Bathurst and Yu (2018)	1.00	0.39	
	Steel grid	c=0, \$\$>0	97	BSI (2010)	1.36	0.50	Miyata and Bathurst
				Simplified method (AASHTO	1.01	0.45	(2019)
				2017)			
				PWRC (2014)	1.41	0.47	
		c≥0,	113	BSI (2010)	1.19	0.64	
				AASHTO (2017)	0.89	0.59	
				PWRC (2014)	1.23	0.63	
		c=0, \$\$>0	97	Coherent gravity method	1.36	0.50	Bathurst et al. (2020)
				(AASHTO 2017)			
				Bathurst and Yu (2018)	1.00	0.32	
				Allen and Bathurst (2015, 2018)	0.99	0.35	
Pull-out	Geosynthetic	Uniaxial HDPE	159	AASHTO (2007)	2.02	0.47	Huang and Bathurst
		Biaxial PP	25		2.68	0.5	(2009)
		Woven PET	134		2.41	0.59	
		All	318		2.23	0.55	
				Huang and Bathurst (2009)	1.07	0.36	
	Geogrid	Gravel to fine sand	194	PWRC (2000)	1.11	0.23	Miyata and Bathurst
		Sand	160		1.28	0.27	(2012c)
		Silty sand	149		1.75	0.37	

0.38 All 503 1.35 Smooth steel strip 47 Berg et al. (2009a, b) 2.73 0.48 Huang et al. (2012) 38 2.5 Ribbed steel strip 0.54 Soil 36 PWRC (1988) Miyata and Bathurst 1.45 0.39 type A(laboratory) (2012b) Soil type A(in situ) 1.42 0.5 128 Soil type B (in situ) 3.27 0.4 43 Cohesionless soil 180 AASHTO (2017) 2.53 0.46 Bozorgzadeh et al. Strip PWRC (2014) 2.34 0.47 (2020)Miyata and Bathurst (2012b) 1.00 0.40 Steel grid Laboratory, c=0, $\phi > 0$ 129 Peterson and Anderson (1980) 0.64 0.87 Miyata et al. (2018) Laboratory, c>0, ϕ >0 2.39 56 0.76 Laboratory, $c \ge 0$, $\phi > 0$ 1.17 185 1.17 In situ, c ≥ 0 , $\phi > 0$ 17 4.06 1.12 Laboratory, c=0, $\phi > 0$ Jewell et al. (1984) 2.49 129 0.77 Laboratory, c>0, ϕ >0 56 12.56 0.78 Laboratory, $c \ge 0$, $\phi > 0$ 5.54 1.31 185 In situ, c ≥ 0 , $\phi > 0$ 24.74 17 1.51 Laboratory, c=0, $\phi > 0$ 129 Nebeshima et al. (1999) 0.79 1.16 Laboratory, c>0, ϕ >0 3.46 0.62 56 Laboratory, $c \ge 0$, $\phi > 0$ 0.95 185 1.86 In situ, c ≥ 0 , $\phi > 0$ 5.39 1.03 17 Laboratory, c=0, $\phi > 0$ 129 Berg et al. (2009a, b) 1.33 0.44 Laboratory, c>0, ϕ >0 56 2.44 0.52 Laboratory, $c \ge 0$, $\phi > 0$ 185 1.67 0.59 In situ, c ≥ 0 , $\phi > 0$ 17 1.53 0.69

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			Laboratory, c=0, \$\$>0	129	Revised Berg et al. (2009a, b)	1.15	0.43	
			Laboratory, c>0, \$\$>0	56		2.16	0.53	
			Laboratory, c≥0, φ>0	185		1.45	0.61	
			In situ, c≥0, φ>0	17		1.35	0.71	
			Laboratory, c=0, \$\phi>0	129	Yu and Bathurst (2015)	1.07	0.34	
			Laboratory, c>0, \$\$>0	56		2.26	0.32	
			Laboratory, c≥0, φ>0	185		1.43	0.52	
			In situ, c≥0, φ>0	17		2.65	0.56	
			Laboratory, c=0, ϕ >0	129	Revised Yu and Bathurst (2015)	1.00	0.34	
			Laboratory, c>0, \$\$>0	56		1.01	0.33	
			Laboratory, c≥0, φ>0	185		1.00	0.34	
			In situ, c≥0, φ>0	17		1.14	0.35	
MAW	Load		c=0, \$\$>0	18	PWRC (2002)	0.98	0.67	Miyata et al. (2009)
			c>0, \$\$>0	18		0.57	0.79	
			c≥0, \$>0	36		0.78	0.76	
	Pull-out		c≥0, \$>0	28	PWRC (2002)	1.21	0.35	Miyata et al. (2011)
SNW	Load	Long-term	All soil	54	AASHTO (2014)	0.95	0.38	Lin et al. (2017a)
		Short-term	All soil	45		0.66	0.52	
			Cohesive	92	CABR (2012)	0.6	1.01	Yuan et al. (2019)
			Cohesionless	52		0.64	0.92	
			All soil	144		0.62	0.97	
			Cohesive	92	CECS (1997)	0.65	0.74	
			Cohesionless	52		0.58	0.71	
			All soil	144		0.62	0.73	
	Pull-out	Hong Kong data	CDG soil	74	GEO (2007)	2.98	0.36	Lin et al. (2017b)
			CDV soil	30		3.58	0.43	

Face tensile	Long term	All facing	42	Lazarte et al. (2015)	0.85	0.43	Liu et al. (2018)
	Short term		23		0.77	0.67	

Note:

- 1. X_m =measured response; X_c =calculated response; M=model factor (= X_m/X_c); N=number of tests or data points; λ =mean (bias) of model factor; B=foundation width; s_u or s_{u0} =undrained shear strength at the surface of the clay layer; D/B=embedment ratio; θ =slope angle; ϕ =friction angle; RMR=rock mass rating; H_s/B=ratio of thickness of sand layer to foundation width; D_r =relative density; ϕ_{cv} =constant volume friction angle of sand; s_m =measured settlement; s_c =calculated settlement; L=length of pipeline; α =foundation base cone angle; σ_c =uniaxial compressive strength of rock; E_m =elasticity modulus of rock mass; GSI=geological strength index; RQD=rock quality designation; Q_{L2} =failure load interpreted by the L₁-L₂ method in Hirany and Kulhawy (1988); PI=plasticity index; OCR=overconsolidation ratio; S_r =soil sensitivity; d=shaft diameter of helical pile; n=number of bearing helices; S/B=ratio of helix spacing to diameter; Q=applied load; L=pile length; A=cross-sectional area of pile; E=elasticity modulus of pile material; h/D=ratio of lateral load eccentricity to shaft depth; Q_{uc} =calculated capacity; Q_{DA} =failure load interpreted by the Davisson (1972) offset limit method; SPT=standard penetration test; ASTM=American Society for Testing and Materials; LL=liquid limit; PL=plasticity limit; FS_A=actual factor of safety; H=slope height or wall height; H_e=excavation depth; H_d=wall embedment depth; B_e=excavation width; CEP=closed-ended pile; CFA=continuous flight auger pile; RC=round concrete; SC=square concrete; SPC=steel pipe closed; SPO = steel pipe open; SNW=soil nail wall; and MSE=mechanically stabilized earth wall.
- 2. The mean (bias) λ and COV values in Akbas (2007) for shallow foundation settlement, Stuyts et al. (2016) for upheaval buckling of offshore pipeline, and Machairas et al. (2018) for driven pile capacity were calculated for the ratio of calculated over measured response.

5. Statistics for transformation uncertainties

Jianye Ching and Ali Noorzad

5.1 Introduction

Transformation models (Phoon and Kulhway 1999) quantify correlation behaviors among various soil/rock parameters. Transformation models are used to infer geotechnical properties from indirect measurements. A site-specific transformation model can be calibrated with direct and indirect measurements from a site. When such a model is used, spatial variability, measurement errors, and statistical uncertainty propagate into the uncertainty of the spatial average, which is the variable of interest in most geotechnical analyses (van der Kroget et al. 2019). Useful compilations of these models are available in the literature (e.g., Kulhawy and Mayne 1990; Mayne et al. 2001; Zhang 2016).

A transformation model is usually developed by regression based on a soil/rock dataset, so it is suitable for the range of conditions found in the dataset. The resulting regression equation does not provide an exact fit to the dataset, and the variability is called the transformation uncertainty. When implemented to a future case that is not within the regression dataset, a transformation model may exhibit both bias and variability. Ching and Phoon (2014) defined the bias (b) and coefficient of variation (COV) (δ) as the sample mean and sample COV of the following ratio:

 $\varepsilon = \frac{\text{measured target value}}{\text{prediction made by a transformation model}}$ (5.1)

The bias (b) and COV (δ) of a transformation model can be calibrated by a generic soil/rock database. Table 5.1 gives the details of some generic soil/rock databases, labelled as (material type)/(number of parameters of interest)/(number of data points). For instance, consider the first transformation model in Table 5.2. The target value is s_u^{re}/P_a (s_u^{re} is the remolded undrained shear strength; P_a is one atmosphere pressure), and the prediction is $0.0144 \times LI^{-2.44}$ (LI is liquidity index). For all cases in the CLAY/10/7490 database (see Table 5.1) with simultaneous knowledge of s_u^{re} and LI, their ϵ ratios can be computed, and b and δ are simply the sample mean and sample COV of these ϵ ratios. In principle, b = 1 is desirable because this means that the prediction is unbiased, whereas $\delta = 0$ is desirable because this means that b×prediction is 100% accurate. In this report, b and δ of some existing transformation models found in the literature are calibrated by the generic soil/rock databases in Table 5.1.

5.2 Summary Tables

Tables 5.2 to 5.5 summarize the calibrated b and δ values of some existing transformation models for clay, sand, rock, and rock mass, respectively. The n value in the third column is the number of calibration cases with simultaneous knowledge of measured value and prediction.

State-of-the-art review of inherent variability and uncertainty, March 2021 **Table 5.2.** Soil/rock databases

Database	Reference	Parameters of interest	# data points	# sites/ studies
CLAY/10/7490	Ching and Phoon (2014)	LL, PI, LI, σ'_{ν}/P_a , σ'_p/P_a , s_u/σ'_v , S_t , q_{t1} , q_{tu} , B_q	7490	251 studies
SAND/7/2794	Ching et al. (2017)	$D_{50}, C_u, D_r, \sigma'_{v}/P_a, \phi', Q_{tn}, (N_1)_{60}$	2794	176 studies
ROCK/9/4069	Ching et al. (2018)	$\gamma,n,R_L,S_h,\sigma_{bt},I_{s50},V_p,\sigma_{ci},E_i$	4069	184 studies
ROCKMass/9/5876	Ching et al. (2020)	RQD, RMR, Q, GSI, E_m , E_{em} , E_{dm} , E_i , σ_{ci}	5784	225 studies
CLAY/8/12225	Ching (2020)	LL, PI, w, e, o'v, Cc, Cs, cv	12225	427 studies
CLAY/12/3997	Ching (2020)	LL, PI, LI, σ_v/P_a , σ_p/P_a , s_u/σ_v , K_0 , E_u/σ_v , B_q , q_{t1} , $N_{60}/(\sigma_v/P_a)$	3997	237 studies
SAND/10/4113	Ching (2020)	e, Dr, σ'_v/P_a , σ'_p/P_a , K ₀ , E _{d1} , Q _{tn} , B _q , (N ₁) ₆₀ , K _{DMT}	4113	172 studies

Note: $\gamma =$ unit weight; $\phi' =$ effective friction angle; $\sigma'_p =$ preconsolidation stress; $\sigma'_v =$ vertical effective stress; $\sigma_{bt} =$ Brazilian tensile strength; $\sigma_{ci} =$ uniaxial compressive strength of intact rock; $(N_1)_{60} = N_{60}/(\sigma'_v/P_a)^{0.5}$; $B_q =$ CPT pore pressure ratio = $(u_2-u_0)/(q_t-\sigma_v)$; $C_c =$ compression index; $C_s =$ swelling index; $C_u =$ coefficient of uniformity; $c_v =$ coefficient of consolidation; $D_{50} =$ median grain size; $D_r =$ relative density; e = void ratio; $E_d =$ drained modulus of sand; $E_{d1} = (E_d/P_a)/(\sigma'_v/P_a)^{0.5}$; $E_{dm} =$ dynamic modulus of rock mass; $E_e =$ elasticity modulus of rock mass; $E_i =$ Young's modulus of intact rock; $E_m =$ deformation modulus of rock mass; $E_u =$ undrained modulus of clay; GSI = geological strength index; $I_{s50} =$ point load strength index; LL = liquid limit; n = porosity; $N_{60} =$ corrected SPT-N; $P_a =$ atmospheric pressure = 101.3 kPa; PI = plasticity index; Q = Q-system; $q_c =$ cone tip resistance; $q_t =$ corrected cone tip resistance; $Q_{tn} = (q_t/P_a)/(\sigma'_v/P_a)^{0.5}$; $q_{t1} = (q_t-\sigma_v)/\sigma'_v =$ normalized cone tip resistance; $q_{tu} = (q_t-u_2)/\sigma'_v =$ effective cone tip resistance; $R_L =$ L-type Schmidt hammer hardness; RMR = rock mass rating; RQD = rock quality designation; $S_h =$ Shore scleroscope hardness; SPT-N = standard penetration test blow count; $S_t =$ sensitivity; $s_u =$ undrained shear strength for clay; $s_u^{re} =$ remoulded s_u ; $u_0 =$ hydrostatic pore pressure; $u_2 =$ CPTU pore pressure; $V_p =$ P-wave velocity; w = water content.

Figure 5.1 shows the calibrated (b, δ) values. According to the classification for model bias proposed by Phoon and Tang (2019), a similar classification is adopted for the transformation bias:

- 1. Highly overestimation (b < 0.5);
- 2. Moderately overestimation $(0.5 \le b < 1)$;
- 3. Moderately underestimation $(1 \le b < 2)$;
- 4. Highly underestimation $(2 \le b < 3)$;
- 5. Very highly underestimation ($b \ge 3$).

According to the classification for model COV proposed by Phoon and Tang (2019), a similar classification is adopted for the transformation COV:

- 1. Low variability ($\delta < 0.3$);
- 2. Medium variability $(0.3 \le \delta < 0.6)$;
- 3. High variability $(0.6 \le \delta < 0.9)$;
- 4. Very high variability ($\delta \ge 0.9$).

5.3 Key observations

- 1. Most transformation models have 0.5 < b < 2 (namely, moderate underestimate or moderate underestimate).
- 2. Most soil models (clay and sand) and intact rock models have $0.3 < \delta < 0.9$ (median to high variability). However, rock mass models have the largest variability. Most of them are with $\delta > 0.9$ (very high variability).
- 3. Most clay models for s_u and OCR (or σ'_p) have $0.3 < \delta < 0.6$ (median variability).
- 4. Most clay models for C_c and C_s have $0.6 < \delta < 0.9$ (high variability).
- 5. Most sand models for ϕ' and D_r have $\delta < 0.3$ (low variability). In particular, all sand models for ϕ' have very low variability ($\delta \approx 0.1$).

- 6. All soil models for modulus (e.g., E_{PMT} , E_{DMT} , and M') have $\delta > 0.6$ (high to very high variability). Moreover, they are fairly biased with 1.5 < b < 2.3 (moderate to high underestimation).
- 7. All soil models for K₀ have $0.3 < \delta < 0.9$ (median to high variability).
- 8. In general, the COVs for transformation uncertainties are significantly larger than those for spatial variability. The latters (COVs for spatial variability) usually range from 0.1 to 0.5.



Figure 5.1. Calibrated (b, δ) values

5.4 References

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Table 5.2. Transformation models for clay	
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Model	Literature	n	Transformation model	b	δ	Data restriction / Application range	Calibration database
$s_u^{re} - LI$	Locat and Demers (1988)	899	$s_{\rm u}^{re}/P_a \approx 0.0144 \times LI^{-2.44}$	1.92	1.25		CLAY/10/7490
$S_t - LI$	Bjerrum (1954)	1279	$S_t \approx 10^{0.8LI}$	2.06	1.09		CLAY/10/7490
	Stas and Kulhawy (1984)	249	$\sigma_p'/P_a \approx 10^{1.11 - 1.62 \times LI}$	2.94	1.90	$S_t < 10$	CLAY/10/7490
$o_p - LI - S_t$	Ching and Phoon (2012b)	489	$\sigma'_p / P_a \approx 0.235 \times LI^{-1.319} \times S_t^{0.536}$	1.32	0.78		CLAY/10/7490
$s_u - \sigma'_p$	Mesri (1975, 1989)	1155	$s_u(mob)/\sigma'_p \approx 0.22$	1.04	0.55		CLAY/10/7490
$\left(\frac{s_u}{\sigma'_v}\right) - OCR$	Jamiolkowski et al. (1985)	1402	$s_u(mob)/\sigma'_v \approx 0.23 \times OCR^{0.8}$	1.11	0.53		CLAY/10/7490
$\left(\frac{s_u}{\sigma'_v}\right) - OCR - S_t$	Ching and Phoon (2012b)	395	$s_u(mob)/\sigma'_v \approx 0.223 \times OCR^{0.823} \times S_t^{0.121}$	0.84	0.34		CLAY/10/7490
$\left(\frac{s_u}{\sigma'_v}\right) - CPT$	Ching and Phoon (2012a)	423	$s_u(mob)/\sigma'_v \approx \frac{(q_t - \sigma_v)/\sigma'_v}{29.1 \times \exp(-0.513B_q)}$	0.95	0.49		CLAY/10/7490
	Chen and Mayne (1996)	690	$\text{OCR} \approx 0.259 \times [(q_t - \sigma_v)/\sigma'_v]^{1.107}$	1.01	0.42		CLAY/10/7490
OCR - CPT	Kulhway and Mayne (1990)	690	OCR $\approx 0.32 \times (q_t - \sigma_v) / {\sigma'}_v$	1.00	0.39		CLAY/10/7490
$\sigma' = C D T$	Chen and Mayne (1996)	690	$\sigma_p'/P_a\approx 0.227\times [(q_t-\sigma_\nu)/P_a]^{1.200}$	0.99	0.42		CLAY/10/7490
$o_p = crr$	Kulhway and Mayne (1990)	690	$\sigma_p'\approx 0.33\times (q_t-\sigma_v)$	0.97	0.39		CLAY/10/7490
C = U	Skempton (1944)	3398	$C_c \approx 0.007 \times (LL - 10)$	1.59	0.90	Remolded clay	CLAY/8/12225
$C_c - LL$	Terzaghi and Peck (1967)	3398	$C_c \approx 0.009 \times (LL - 10)$	1.24	0.90		CLAY/8/12225
$C_c - PI$	Kulhawy and Mayne (1990)	2964	$C_c \approx PI/73$	1.31	0.91		CLAY/8/12225
$C_c - LL - G_s$	Nagaraj and Murty (1985)	1523	$C_c \approx 0.2343 \times (LL/100) \times G_s$	1.06	0.81		CLAY/8/12225
$C_s - e_0$	Peck and Reed (1954)	668	$C_s \approx 0.208 \times e_0 + 0.0083$	0.31	0.69		CLAY/8/12225
$C_s - \omega$	Azzouz et al. (1976)	771	$C_s \approx 0.003 \times (\omega + 7)$	0.47	0.67		CLAY/8/12225

$C_s - PI$	Kulhawy and Mayne (1990)	847	$C_s \approx PI/385$	0.96	0.63	CLAY/8/12225
$C_s - LL$	Isik (2009)	846	$C_s \approx 0.0007 \times \text{LL} + 0.0062$	1.55	0.64	CLAY/8/12225
$K_0 - OCR$	Mayne and Kulhawy (1990)	1009	$K_0 \approx [1 - \sin(\phi')] \times OCR^{\sin(\phi')}$	1.00 (\phi' = 25°)	0.34 (\phi' = 25°)	CLAY/12/3997
$K_0 - K_{DMT}$	Wayne and Kunawy (1990)	124	$K_0 \approx 0.27 \times K_{DMT}$	0.92	0.45	CLAY/12/3997
	Powell and Uglow (1988)	124	$K_0 \approx 0.34 \times K_{DMT}^{0.55}$	1.22	0.30	CLAY/12/3997
$K_0 - N_{60} - \sigma'_{\nu}$	Kulhowy et al. (1090)	74	$K_0 \approx 0.073 \times N_{60}/(\sigma'_{\nu}/P_a)$	2.16	0.70	CLAY/12/3997
$K_0 - CPT$	Kulliawy et al. (1989)	74	$K_0 \approx 0.1 \times (q_t - \sigma_v) / {\sigma'}_v$	1.18	0.74	CLAY/12/3997
$E_{PMT} - N_{60}$	Kulhway and Mayne (1990), Ohya et al. (1982)	812	$E_{PMT}/P_a \approx 19.3 \times N_{60}^{0.63}$	1.60	1.28	CLAY/12/3997
M' - CPT	Mayne (2007)	111	$M' \approx 5 \times (q_t - \sigma_v)$	1.57	0.75	CLAY/12/3997

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Note: $s_u(mob) =$ undrained shear strength mobilized in embankment and slope failures (Mesri and Huvaj 2007); $E_{PMT} =$ pressuremeter modulus; M' = effective constrained modulus.

Table 5.3. Transformation models for sand

Model	Literature	n	Transformation model	b	δ	Data restriction / Application range	Calibration database
	Terzaghi and Peck (1967)	198	$D_r(\%) \approx 100 \times \sqrt{(N_1)_{60}/60}$	1.05	0.23	$(N_1)_{60} < 60$	SAND/7/2794
$D_r - SPT$	Kulhawy and Mayne (1990)	199	$D_r(\%) \approx 100 \times \sqrt{\frac{(N_1)_{60}}{[60 + 25 \log_{10}(D_{50})] \times OCR^{0.18}}}$	1.01	0.21		SAND/7/2794
$D_r - CPT$	Jamiolkowski et al. (1985)	681	$D_r(\%) \approx 68 \times [\log_{10}(Q_{tn}) - 1]$	0.84	0.33	$Q_{tn} < 300$	SAND/7/2794
$\phi' - D_r$	Bolton (1986)	391	$\phi' \approx \phi'_{CV} + 3 \times \left(D_r \left[10 - \ln \left(p'_f \right) \right] - 1 \right)$	1.03	0.052		SAND/7/2794
$\phi' - SPT$	Hatanaka et al. (1998)	58	$\phi' \approx \begin{cases} \sqrt{15.4 \times (N_1)_{60}} + 20 & (N_1)_{60} \le 26 \\ 40 & (N_1)_{60} > 26 \end{cases}$	1.07	0.090	$(N_1)_{60} < 150$	SAND/7/2794

	Chen (2004)	59	$\phi' \approx 27.5 + 9.2 \times \log_{10} [(N_1)_{60}]$	1.00	0.095	SAND/7/2794
$\phi' - CPT$	Robertson and Campanella (1983)	99	$\phi' \approx \tan^{-1}[0.1 + 0.38 \times \log_{10}(q_t/\sigma'_v)]$	0.93	0.056	SAND/7/2794
	Kulhawy and Mayne (1990)	376	$\phi' \approx 17.6 + 11 \times \log_{10}(Q_{tn})$	0.97	0.081	SAND/7/2794
$E_{PMT} - N_{60}$	Kulhway and Mayne (1990), Ohya et al. (1982)	1081	$E_{PMT}/P_a \approx 9.08 \times N_{60}^{0.66}$	2.24	1.05	SAND/10/4113
$E_{DMT} - N_{60}$	Mayne and Frost (1989)	591	$E_{PMT}/P_a \approx 22 \times N_{60}^{0.82}$	1.69	0.72	SAND/10/4113
M' - CPT	Mayne (2007)	113	$M' \approx 5 \times (q_t - \sigma_v)$	1.57	0.76	SAND/10/4113
$K_0 - OCR$	Mayne and Kulhawy (1990)	1207	$K_0 \approx [1 - sin(\phi')] \times OCR^{sin(\phi')}$	1.05 (\phi' = 30°)	0.50 (\phi' = 30°)	SAND/10/4113

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Note: E_{DMT} = pressuremeter modulus; E_{PMT} = pressuremeter modulus; M' = effective constrained modulus.

Model	Literature	n	Transformation model	b	δ	Data restriction / Application range	Calibration database
$\sigma_{ci} - n$	Kilic and Teymen (2008)	911	$\sigma_{ci}\approx 147.16\times e^{-0.0835n}$	0.91	0.75	0.16 < n < 37.81	ROCK/9/4069
$\sigma_{ci} - R_L$	Karaman and Kesimal (2015)	664	$\sigma_{ci} \approx 0.1383 \times R_L^{1.743}$	0.78	0.53	$11.82 < R_L < 59.59$	ROCK/9/4069
$\sigma_{ci} - S_h$	Altindag and Guney (2010)	297	$\sigma_{ci} \approx 0.1821 \times S_h^{1.5833}$	1.15	0.65	$9 < S_h < 100$	ROCK/9/4069
$\sigma_{ci} - \sigma_{bt}$	Prakoso and Kulhawy (2011)	525	$\sigma_{ci} \approx 7.8 \times \sigma_{bt}$	1.31	0.50	$5 < \sigma_{bt} < 25.64$	ROCK/9/4069
$\sigma_{ci} - I_{s50}$	Mishra and Basu (2013)	1074	$\sigma_{ci} \approx 14.63 \times I_{s_{50}}$	1.18	0.45	$1.15 < I_{s_{50}} < 14.13$	ROCK/9/4069
$\sigma_{ci} - V_P$	Kahraman (2001)	1247	$\sigma_{ci} \approx 9.95 \times V_P^{1.21}$	1.26	0.63	$1.02 < V_P < 6.3$	ROCK/9/4069
$E_i - R_L$	Katz et al. (2000)	289	$E_i \approx 0.00013 \times R_L^{3.09074}$	1.47	0.99	$24.01 < R_L < 73.3$	ROCK/9/4069
$E_i - S_h$	Deere and Miller (1966)	197	$E_i \approx 0.739 \times S_h + 11.51$	0.61	0.71	$11 < S_h < 105$	ROCK/9/4069
$E_i - \sigma_c$	Deere and Miller (1966)	1152	$E_i \approx 0.303 \times \sigma_{ci} - 0.8745$	1.23	0.94	$21 < \sigma_{ci} < 1330$	ROCK/9/4069
$E_i - V_P$	Yasar and Erdogan (2004)	192	$E_i \approx 10.67 \times V_P - 18.71$	0.90	0.72	$3.11 < V_P < 5.6$	ROCK/9/4069

 Table 5.4. Transformation models for intact rock

Note: σ_{ci} in MPa; E_i in GPa; I_{s50} in MPa; σ_{bt} in MPa; V_P in km/s; n in %.

Model	Literature	n	Transformation model	b	δ	Data restriction / Application range	Calibration database
	Coon and Merritt (1970)	147	$E_m/E_i = 0.0231 \times RQD - 1.32$	1.26	1.09	57 < RQD < 100	ROCKMass/9/5876
$E_m - E_i - RQD$	Zhang and Einstein (2004)	161	$E_m/E_i = 10^{0.0186 \times RQD - 1.91}$	1.54	0.89	0 < RQD < 100	ROCKMass/9/5876
	Bieniawski (1978)	1091	$E_m = 2 \times RMR - 100$	0.57	1.48	50 < RMR < 85	ROCKMass/9/5876
$E_m - RMR$	Gokceoglu et al. (2003)	1749	$E_m = 0.0736 \times e^{0.0755 \times RMR}$	1.53	1.21	20 < RMR < 85	ROCKMass/9/5876
	Serafim and Pereira (1983)	1749	$E_m = 10^{\frac{(RMR-10)}{40}}$	0.51	1.00	20 < RMR < 85	ROCKMass/9/5876
$E_m - Q$	Grimstad and Barton (1993)	288	$E_m = 25 \times \log_{10}(Q)$	0.58	0.88	1.1 < Q < 1000	ROCKMass/9/5876
$E_m - GSI$	Hoek and Diederichs (2006)	349	$E_m = 100 \times \frac{(1 - D/2)}{1 + e^{\frac{75 + 25D - GSI}{11}}}$	0.83 (D = 0)	1.00 (D = 0)	10 < GSI < 100	ROCKMass/9/5876

 Table 5.5. Transformation models for rock mass

Note: σ_{ci} in MPa; E_m in GPa; D is disturbance factor (Hoek and Diederichs 2006).

6. Determining characteristic values of geotechnical parameters and resistance: an overview

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6.1 Introduction

Determining design values of geotechnical parameters or resistance is indispensable in geotechnical design practice. Depending on design approaches adopted, the design values of geotechnical parameters and resistance are often derived from their respective characteristic values. For example, when partial material factor design methods (e.g., Design Approach 3 in Eurocode 7) are applied, characteristic values of geotechnical parameters are needed while characteristic values of geotechnical resistance (e.g., pile compressive resistance) are used in resistance factor design methods (e.g., Design Approach 2 in Eurocode 7) (CEN 2004; Orr 2017). Selection of characteristic values of geotechnical parameters and resistance is an intriguing issue and has attracted much attention (e.g., Schneider 1997; Hicks and Samy 2002; Honjo 2009; Schneider and Schneider 2013; Hicks 2013; Orr 2000, 2017; Wang et al. 2017; Zhao et al. 2018; Shen et al. 2019; Hicks et al. 2019; Prästings et al. 2019; van der Krogt et al. 2019; Ching et al. 2020; Tabarroki et al. 2020; Länsivaara et al. 2020). Despite these efforts, the ways of determining the characteristic values of geotechnical parameters and resistance remain debatable. A diversity of definitions and selection methods of characteristic values of geotechnical parameters and resistance can be found in the geotechnical literature and national (or regional) design codes with consideration of various factors. For example, in Eurocode 7, Clause 2.4.5.2(4)P lists the following six factors that shall be taken into account for selection of characteristic values of geotechnical parameters:

- geological and other background information, such as data from previous projects;(F1)
- •the variability of the measured property values and other relevant information, e.g. from existing knowledge; (F2)
- •*the extent of the field and laboratory investigation; (F3)*
- •*the type and number of samples; (F4)*
- •*the extent of the zone of ground governing the behaviour of the geotechnical structure at the limit state being considered; (F5)*
- •the ability of the geotechnical structure to transfer loads from weak to strong zones in the ground. (F6)

Note that the "F1"-"F6" in parentheses following each item are added by authors of this report for the convenience of discussions. The above six factors include aspects on prior knowledge (F1 and F2), spatial variability and statistical uncertainties (F2 and F3), data quantity and quality (F4), and limit state & response of the geotechnical structure (F5 and F6). Although some of these aspects may have been considered in other national (or regional) geotechnical codes as well, definitions of

characteristic values are often worded to avoid being too prescriptive, allowing geotechnical engineers to exercise their engineering judgment and experience to select characteristic values at a site in a subjective way. Nevertheless, it has been observed that such a subjective reasoning might result in a wide spread of selected characteristic values for a given set of site investigation data (Bond and Harries 2008; Orr 2017). Differences between engineers' subjective estimates of characteristic values can be attributed to differences in their level of expertise obtained from their educational background, differences in deliberate practices, as well as various cognitive biases and limitations (Vick 2002; Cao et al. 2016).

Alternatively, statistical methods can be employed in selection of characteristic values of geotechnical parameters and resistance. Several statistical methods have been developed in the literature. To facilitate the understanding and selection of these methods, this report presents an overview on existing statistical methods of determining characteristic values of geotechnical parameters and resistance. The overview covers both frequentist and Bayesian formulas to calculate characteristic values of geotechnical parameters. Frequentist and Bayesian schools assert different interpretations of probability, i.e., relative frequency versus degrees-of-belief (e.g., Vrouwevelder 2002; Baecher and Christian 2003). This leads to the use of different probabilistic terminologies in practice, such as "confidence interval" in frequentist approaches versus Bayesian "credibility interval". Nevertheless, this report will not strictly pursue the implication of the difference in frequentistic and Bayesian probability axioms. This allows interchangeably implementing frequentist and Bayesian approaches in practice, e.g., statistics estimated from Bayesian approaches can be used in frequentist formulas to calculate the characteristic values.

6.2 Definition of characteristic values of geotechnical parameters and resistance

Tables 6.1 and 6.2 summarize, respectively, definitions of characteristic values of geotechnical parameters and resistance in geotechnical design codes (including Eurocodes, CHBDC in Canada, ASSHTO LRFD BDS in USA, Australia Standard, and CIGE in China) and the literature. A glance reveals that these definitions are far from a clear cut. Canadian Foundation Engineering Manual (CFEM) (CGS 2006), AASHTO LRFD BDS (i.e., AASHTO 2017), Australian Standard AS5100.3 (SA 2004), and CIGE (MOHURDPRC 2009) incorporate the conservatism into selection of characteristic values, X_k , of geotechnical parameters. Eurocode 7 (CEN 2004) and CHBDC 2019 (CSA 2019) share a similar definition of X_k that considers both the conservatism (i.e., "*a cautious estimate*") and physical mechanism (i.e., "*affecting the occurrence of the limit state*" and "*within the zones of influence of applied loads*"). Moreover, Eurocode 7 distinguishes two scenarios involving global and local failures, for which the zones of ground affect the behavior of geotechnical structures at the limit state are of different sizes with respect to the scale of fluctuation (SOF) of geotechnical parameters (e.g., Frank 2004; Orr 2017; Prästings et al. 2019). For the global failure scenario with a size of the influence zone much greater than the SOF, the X_k should be a cautious

State-of-the-art review of inherent variability and uncertainty, March 2021 estimate of the mean value within the influence zone affecting the occurrence of limit state. The influence zone can be a surface or volume of ground, depending on the design problems concerned. For the local failure scenario with a size of the influence zone smaller than the SOF, the X_k should be taken as a cautious estimate of the lowest or highest value within the influence zone. These considerations highlight the significant role of geotechnical spatial variability and its interaction with ground mechanical responses in selection of characteristic geotechnical parameters.

Sources	Clause	Definition
EN 1997-1 (CEN	2.4.5.2(4)P	The characteristic value of a geotechnical parameter shall be selected as a
2004)		cautious estimate of the value affecting the occurrence of the limit state.
	2.4.5.2(11)	If statistical methods are used, the characteristic value should be derived
		such that the calculated probability of a worse value governing the
		occurrence of the limit state under consideration is not greater than 5%.
		NOTE In this respect, a cautious estimate of the mean value is a selection
		of the mean value of the limited set of geotechnical parameter values,
		with a confidence level of 95%; where local failure is concerned, a
		cautious estimate of the low value is a 5% fractile.
CHBDC 2019	6.2	Characteristic geotechnical parameter $-$ a cautious estimate of the
(CSA 2019)		mean value of a geotechnical parameter for individual strata within the
		zones of influence of applied loads.
CFEM (CGS		Frequently, the mean value, or a value slightly less than the mean is
2006)		selected by geotechnical engineers as the characteristic value.
AS5100.3		the characteristic value of a geotechnical parameter should be a
(SA 2004)		conservatively assessed value of the parameter
AASHTO LRFD	C10.4.6.1	For strength limit states, average measured values of relevant laboratory
BDS (AASHTO		test data, in situ test data, or both were used to calibrate the resistance
2017)		factorsit may not be possible to reliably estimate the average value of
		the properties needed for design. In such cases, the Engineer may have no
		choice but to use a more conservative selection of design parameters to
		mitigate the additional risks created by potential variability or the paucity
		of relevant data.
CIGE	2.1.13	Representative value of geotechnical parameters and it usually taken as
(MOHURDPRC		the 5% quantile value.
2009)		

Table 6.1. Definition of characteristic values of geotechnical parameters

Sources	Clause	Definition
EN 1997-1	7.6.2.2(7)P	When deriving the ultimate characteristic compressive resistance $R_{c;k}$ from
(CEN 2004)		values $R_{c;m}$ measured in one or several pile load tests, an allowance shall be
		made for the variability of the ground and the variability of the effect of
		pile installation.
	8.4(10)P	For grouted anchorages and screw anchorages, the characteristic value of
		the pull-out resistance, $R_{a;k}$, shall be determined on the basis of suitability
		tests according to 8.7 or comparable experience. The design resistance
		shall be checked by acceptance tests after execution.
	8.5.2(3)	The characteristic value should be related to the suitability test results by
		applying a correlation factor ξ_a .
		NOTE 8.5.2(3) refers to those types of anchorage that are not individually
		checked by acceptance tests. If a correlation factor ξ_a is used, it must be
		based on experience or provided for in the National annex.
CHBDC 2019	6.2	Serviceability geotechnical resistance — the load that the ground can
(CSA 2019)		support at serviceability limit states, usually at a predefined deformation in
		the ground or structure, estimated using characteristic geotechnical
		parameters; Ultimate geotechnical resistance — the maximum load that
		the ground can support at ultimate limit states, estimated using
		characteristic geotechnical parameters.
NBCC User's	Commentary	the [characteristic] resistance is the engineer's best estimate of the ultimate
Guide (NRC	Κ	resistance
2011)		
NCHRP Report		The nominal values (e.g., the nominal strength, R_n) are those calculated by
507(Paikowsky		the specific calibrated design method and are not necessarily the means
et al. 2004)		(i.e., the mean loads or mean resistance (Figure 1).
AASHTO	10.6.3.1.2a	The nominal bearing resistance shall be estimated using accepted soil
LRFD BDS		mechanics theories and should be based on measured soil parameters. The
(AASHTO		soil parameters used in the analyses shall be representative of the soil shear
2017)		strength under the considered loading and subsurface conditions
	10.6.3.2.2	The nominal bearing resistance of rock should be determined using
		empirical correlation with the Geomechanics RMR system. Local
		experience shall be considered in the use of these semi-empirical
		procedures.
	10.6.3.2.3	The nominal bearing resistance of foundations on rock shall be determined
		using established rock mechanics principles based on the rock mass
		strength parameters. The influence of discontinuities on the failure mode
		shall also be considered.

State-of-the-art review of inherent variability and uncertainty, March 2021 **Table 6.2.** Definition of characteristic values of geotechnical resistance

Characteristic values of geotechnical resistance are needed in resistance factor design approaches (e.g., Design Approach 2 in Eurocode 7 and LRFD in North America). It shall be pointed out that, in some design codes (e.g., ASSHTO LRFD BDS), the terminology "nominal resistance" is used in lieu of "characteristic resistance" while it is also included in this report because it plays the same role of "characteristic resistance" in design calculations. Fenton et al. (2016) summarized definitions of characteristic geotechnical resistance in geotechnical design codes, including Eurocodes, CHBDC, NBCC, ASSHTO LRFD BDS, and Australian Standard. In general, with design calculation models, the characteristic geotechnical resistance can be calculated based on characteristic values of geotechnical parameters. Fenton et al. (2016) observed different degrees of conservatism in characteristic geotechnical resistance adopted in Eurocodes and North American design codes. Eurocode 7 also allows deriving characteristic values of pile resistances from load test results and soil test profiles and pull-out resistance of anchors from the suitability test results by applying correlation factors (e.g., Bauduin 2002; Orr 2017).

6.3 Methods of determining characteristic values of geotechnical parameters

An overview of existing statistical methods of determining characteristic values of geotechnical parameters give two observations:

- •Different methods may be developed based on probability distributions of different random variables, such as the basic variable X (e.g., soil shear strength parameters), statistical mean (X_m) , spatial average (X_A) , and effective (or mobilized) property (X_E) of X within influence zones;
- •Sophistication of different methods varies in a wide spread to account for different uncertainty sources, as discussed below.

6.3.1 Characteristic value based on the probability distribution of the basic random variable

Assume that the basic random variable X that represents an uncertain geotechnical parameter follows a Gaussian distribution. If Gaussian distribution parameters can be determined based on unlimited test results, the X_k can be calculated as:

MP1:
$$X_k = \mu_X - N_{95}\sigma_X = \mu_X(1 - N_{95}V_X)$$
 (6.1)

where μ_X , σ_X , and $V_X (=\sigma_X/\mu_X)$ are the mean value, standard deviation, and coefficient of variation of *X*, respectively; and $N_{95} = 1.645$. Eq. (6.1) gives the point estimate of 5% quantile of the Gaussian random variable *X* as the characteristic value. It accounts for the uncertainty in estimated value of *X* in a gross manner. Neither statistical uncertainty nor spatial averaging are considered by Eq. (6.1). In this report, methods for determining characteristic values of geotechnical parameters and resistance are denoted by "MP#" and "MR#", respectively, which will not be defined again in the remaining part of this report. Moreover, most of the methods summarized in this report provide formulas to calculate the characteristic values based on the Gaussian assumption. When the

coefficient of variation is large, it is more appropriate to assume the lognormal distribution to avoid negative values. It is straightforward to extend Gaussian-based formulas to obtain formulas for the lognormal distribution. This report will give lognormal counterparts of some methods for illustration, but not provide all of them for the sake of conciseness.

6.3.2 Characteristic value considering the statistical uncertainty

With a limited number of test data, it is impossible to determine the statistics (i.e., mean value and standard deviation) of *X* with complete certainty. Statistical uncertainty is unavoidable. Hence, the statistics (e.g., mean value X_m) of *X* are random variables themselves. Depending on whether the standard deviation (i.e., σ_X) of *X* is known or not, a different type of distribution should be adopted (JGS 2004; DNV 2010; ISO2394 2015). When the σ_X is known or assumed based on the prior knowledge, the normal distribution is adopted. For the global failure scenario, the X_k can be taken as the 5% factile of X_m :

MP2:
$$X_k = X_m - N_{95} \frac{\sigma_X}{\sqrt{n}} = X_m (1 - N_{95} \frac{V_X}{\sqrt{n}})$$
 (6.2a)

where *n* is the number of test data. For the local failure scenario, the X_k can be taken as the 5% factile of *X* with the consideration of the statistical uncertainty in X_m , i.e.,

MP3:
$$X_k = X_m - N_{95} \sqrt{1 + \frac{1}{n}} \sigma_X = X_m (1 - N_{95} \sqrt{1 + \frac{1}{n}} V_X)$$
 (6.2b)

When the σ_X is unknown, the Student's *t*-distribution is adopted. Similarly, for the global failure scenario, the X_k can be taken as the 5% factile of X_m :

MP4:
$$X_k = X_m - t_{95,n-1} \frac{S_X}{\sqrt{n}} = X_m (1 - t_{95,n-1} \frac{V_X}{\sqrt{n}})$$
 (6.3a)

where S_X is the sample estimate of standard deviation; $t_{95,n-1}$ is a Student's-*t* factor evaluated for a 95% confidence level and *n*-1 degrees of freedom. For the local failure scenario, the X_k can be taken as the 5% factile of *X* with the consideration of the statistical uncertainty in X_m , i.e.,

MP5:
$$X_k = X_m - t_{95,n-1} \sqrt{1 + \frac{1}{n}} S_X = X_m (1 - t_{95,n-1} \sqrt{1 + \frac{1}{n}} V_X)$$
 (6.3b)

Although the X_k given by MP2 and MP4 are defined based on the distribution of X_m with a 95% confidence level, different confidence levels can be applied to represent different degrees of conservatism. For example, in Russian Code of Practice (e.g., SP 22.13330.2016; SP 23.13330.2018), 95% confidence level is used to selecting characteristic values for design verification of ultimate limit states using MP4 while 85% confidence level is adopted for serviceability limit states.

For considerations of practical implementations, Schneider (1997) proposed the following simple formula based on MP2 and MP4:

MP6:
$$X_k = X_m (1 - \frac{V_X}{2})$$
 (6.4)

The MP6 generally provides a good, but slightly conservative, approximation to MP2 for n > 10 and MP4 for n > 13. Similarly, CIGE of China (MOHURDPRC 2009) provides another approximate formula of MP4 to avoid the misuse of Student's-*t* factor when *n* is relatively small:

MP7:
$$X_k = X_m [1 - (\frac{1.704}{\sqrt{n}} + \frac{4.678}{n^2})V_X]$$
 (6.5)

MP7 gives the same X_k value as MP6 for n = 13. MP2-MP7 account for the statistical uncertainty in X_m , which is indicated by the number, n, of test results in the formulas. For illustration, Figure 6.1 compares characteristic values calculated from MP1-MP7 based on prescribed values of $\mu = 30$, $\sigma_X = 6$, $X_m = 27.68$, $S_X = 7.37$, $V_X = 0.27$, and n = 16, among which X_m , S_X , and V_X are estimated from 16 random samples of a Gaussian random variable with $\mu = 30$ and $\sigma_X = 6$. It is shown that the MP2, MP4, MP6, and MP7 for the global failure scenario give higher characteristic values than MP3 and MP5 for the local failure scenario and MP1 with unlimited test results.

While spatial variability and averaging will be the focus of the methods discussed in the next section, it is important to notice that spatial averaging is already involved in the methods eluded on hitherto. The implicit assumption for the global failure scenario addressed by MP2 and MP4 (and to some degree MP6 and MP7) is that all spatial variability averages. Hence, for the global failure scenario, only the statistical uncertainty in the spatial average is considered. For the local failure scenario addressed by MP3 and MP5, on the other hand, the implicit assumption is that no averaging at all occurs, and that the lowest ground property value in the considered volume governs the failure behavior of the geotechnical structure. These two extremes are in fact the upper and lower bound cases for the more sophisticated treatment of spatial averaging discussed in the next section.



Figure 6.1. Comparison of characteristic values calculated from MP1-MP7

6.3.3 Characteristic value based on the probability distribution of the spatial average

The size of influence zones affecting geotechnical structure responses can be much larger than that involved in-situ and/or laboratory tests. It is more appropriate to consider the spatial average of geotechnical parameters over the influence zone of geotechnical structure responses than their values at a point (or testing) level (Vanmarcke 1977). The estimate of the spatial average, X_A , of X may also be affected by other uncertainty sources arising during site investigation, such as measurement errors, statistical uncertainty, and transformation uncertainty. Its total coefficient of variation, V_{TOT} , can be written as (Phoon and Kulhawy 1999):

$$V_{TOT} = \sqrt{\Gamma_I^2 V_I^2 + V_M^2 + V_T^2 + V_S^2}$$
(6.6)

where Γ_I^2 is the variance reduction function quantifying the extent of reduction in variance due to spatial averaging (Vanmarcke 1977); V_I , V_M , V_T , and V_S are coefficients of variation representing inherent spatial variability, measurement errors, transformation uncertainty, and statistical uncertainty, respectively. Note that Eq. (6.6) is based on an additive uncertainty model, by which the total uncertainty is attributed to inherent spatial variability ε_I , measurement errors ε_M , transformation uncertainty ε_T , and statistical uncertainty ε_S , and these uncertainties are assumed to be independent. Herein, the definition of V_I , V_M , V_T , and V_S is non-traditional in the sense that they are defined as the ratio of their respective standard deviations over the mean value of X, rather than their respective mean values that are usually equal to zero. Moreover, when calibrating a design code, the code writer shall define which uncertainty sources are included in characteristic values and which are considered by partial factors. For example, if V_T has been considered in calibration of partial factors, it can be set to zero in Eq. (6.6), to avoid accounting for it more than once during the design process. For a separable correlation function, the Γ_I^2 can be written as (e.g., Vanmarcke 2010; Schneider and Schneider 2013; Orr 2017):

$$\Gamma_I^2 = \Gamma_X^2 \Gamma_Y^2 \Gamma_Z^2 \tag{6.7}$$

where Γ_i^2 , i = X, Y, Z are the respective variance reduction functions in the two horizontal directions (X and Y) and the vertical direction (Z), and the Γ_i^2 can be approximately calculated as (Schneider and Schneider 2013; Orr 2017):

$$\Gamma_i^2 = \left(\delta_i / L_i\right) \left[1 - \left(\delta_i / 3L_i\right)\right] \quad \text{for} \quad L_i > \delta_i \tag{6.8}$$

$$\Gamma_i^2 = 1 - \left(\delta_i / 3L_i\right) \quad \text{for} \quad L_i \le \delta_i \tag{6.9}$$

where δ_i , i = X, Y, Z is the scale of fluctuation (SOF) in the *i* direction; L_i is the length of the influence zone along the *i* direction.

Depending on the value of V_{TOT} , Schneider and Schneider (2013) suggested the normal and lognormal distributions of X_A , based on which the X_k can be calculated as the 5% quantile of X_A :

MP8:
$$X_k = X_m (1 - N_{95} V_{TOT})$$
 for $V_{TOT} < 0.3$, Normal distribution (6.10a)

MP9:
$$X_k = X_m \cdot \frac{0.2^{\sqrt{\ln(1+V_{TOT}^2)}}}{\sqrt{1+V_{TOT}^2}}$$
 for $V_{TOT} \ge 0.3$, Lognormal distribution (6.10b)

As indicated by Eqs. (6.8)-(6.10), MP8 and MP9 account for not only various uncertainties in estimated X_A (i.e., F2 and F3) but also "the extent of the zone of ground governing the behavior of the geotechnical structure at limit state" (i.e., F5). Van der Krogt et al. (2019) observed that Eq. (6.6) assumes that the measurement errors, statistical uncertainty, and transformation uncertainty are systematic errors that are not subject to spatial averaging. This assumption is conservative since it leads to overestimation of V_{TOT} . They also demonstrated that considering the spatial averaging of measurement errors and transformation uncertainty may lead to further reduction in uncertainty, resulting in a higher characteristic value. It is noteworthy that Ching et al. (2016) presented some real cases showing that transformation uncertainty is indeed not subjected to spatial averaging.

By ignoring measurement errors, statistical uncertainty, and transformation uncertainty, MP8 and MP9 can be simplified as:

MP10:
$$X_k = X_m \left(1 - N_{95} V_I \Gamma_I \right)$$
 for $V_I < 0.3$, Normal distribution (6.11a)

MP11:
$$X_k = X_m \frac{0.2^{\sqrt{\ln(1+V_I^2 \Gamma_I^2)}}}{\sqrt{1+V_I^2 \Gamma_I^2}}$$
 for $V_I \ge 0.3$, Lognormal distribution (6.11b)

Similarly, an evolution committee of CEN proposed another simplified formula (Orr 2017):

MP12:
$$X_{\rm k} = X_m - a(X_m - X_{extr})\sqrt{\delta_Z / L_Z}$$
 (6.12)

where X_{extr} is the expected extreme value of X given a large number of tests; *a* can be taken as 0.5, 0.75, and 1.0, reflecting the extent and quality of field and laboratory investigations or estimation method, type of tests for selecting derived values, and sampling methods and levels of experience (Orr 2017). Assume the X_{extr} is smaller than X_m by 3 times of standard deviation, and δ_Z is taken as a typical value of 1m. Then, MP12 is re-written as:

MP13:
$$X_{\rm k} = X_m - 3aS_X \sqrt{1/L_Z} = X_m (1 - 3aV_X \sqrt{1/L_Z})$$
 (6.13)

Shen et al. (2019) performed a comparative study on MP1, MP4, MP6, MP11, and MP13 from a perspective of design robustness. Based on the comparative study, the following formula for calculating X_k was proposed:

MP14:
$$X_k = X_m (1 - AV_X)$$
 (6.14)

Shen et al. (2019) commented that the optimal value of A in Eq. (6.14) depends on many factors, such as the preference in a trade-off consideration (e.g., cost versus robustness), the cost function in local practice, the level of variation of the noise factors (including the effect of spatial variability), and the "calculated risk", and they suggested a value of a = 0.7, with which Eq. (6.14) has the same format as MP6 suggested by Schneider (1997), but more conservative.

6.3.4 Characteristic value based on the Bayesian theory

All the above formulas for the characteristic value are frequentist ones. Ching et al. (2021) derive the Bayesian formulas for the characteristic value. The derivations assume that both μ_X and σ_X^2 are unknown and follow a prior probability density function (PDF) that is normal-inverse-gamma. If the prior PDF is non-informative, Ching et al. (2021) show that:

MP15:
$$X_k = X_m \left(1 - t_{95,n-1} \sqrt{\Gamma_I^2 V_I^2 + V_S^2 + V_T^2 + V_M^2 / n} \right)$$
 for normal distribution (6.15a)

MP16:
$$X_{k} = \left(\prod_{i=1}^{n} X_{i}\right)^{1/n} \times e^{-t_{95,n-1}\sqrt{\Gamma_{i}^{2}V_{i}^{2} + V_{S}^{2} + V_{I}^{2} + V_{M}^{2}/n}} \qquad \text{for lognormal distribution}$$
(6.15b)

Herein, it is necessary to distinguish V_X and V_I in this report. The V_I denotes the coefficient of

variation of the inherent spatial variability, but the V_X in MP1-MP7, MP13, and MP14 represents the coefficient of variation of X estimated from data without distinguishing different uncertainty sources. Moreover, the V_{TOT} is the coefficient of variation of the total uncertainty with explicit quantification of different uncertainty sources (i.e., ε_I , ε_M , ε_T , and ε_S).

6.3.5 Characteristic value based on responses of geotechnical structures in spatial variability

In terms of the ultimate limit state (ULS), effects of spatial variability on the response of the geotechnical structure are two-fold: the spatial averaging effect that results in the uncertainty reduction, and the weak-path effect that seeks out the failure mechanism with the least resistance and causes the reduction in the mean value of the effective or mobilized shear strength (Hicks and Samy 2002; Hicks 2013; Ching and Phoon 2013; Hu and Ching 2015; Ching et al. 2016, 2017). The effective or mobilized property, X_E , is defined as the homogeneous property that gives the same response (e.g., factor of safety, *FS*) as the response with explicit modelling of the spatial variability. The former factor (spatial averaging) has been considered by MP8-MP13, MP15 and MP16, but the latter factor (weak path) has not been considered. Hicks and Samy (2002) and Hicks (2013) proposed to select geotechnical characteristic parameters based on the random finite element method (RFEM) (Fenton and Griffiths 2008), which is referred to as the "MP17" in this report. The MP17 provides reliability-based characteristic values either based on the probability distribution of X_E or, equivalently, by back-calculating the quantile of *X* from 5% fractile of *FS* determined from the RFEM. Detailed implementation procedures of MP17 are not repeated here, which can be referred to Hicks and Samy (2002), Hicks (2013), and Hicks et al. (2019).

The MP17 successfully considers both the spatial averaging effect and the weak-path effect. Nevertheless, it requires RFEM, which is not trivial for geotechnical practitioners. To address this issue, Ching and Phoon (2013), Hu and Ching (2015), and Ching et al. (2016, 2017) proposed a weakest-path model (WPM) to model X_E . Based on the WPM, the X_E is modeled as the minimum of the X_A values along a number, n_s , of independent representative potential slip curves (RPSCs), i.e., $X_E = min(X_{A,1}, X_{A,2}, ..., X_{A,ns})$, where $X_{A,i}$, $i = 1, 2., n_s$ denotes the spatial average of X along the *i*-th RPSC. However, Ching and Phoon (2013), Hu and Ching (2015), and Ching et al. (2016, 2017) called X_E as the mobilized strength rather than the effective strength, although the two terms are equivalent. Tabarroki et al. (2020) further calibrated the WPM by X_E values simulated by RFEM and developed the simplified formula for the 5% fractile of X_E , defined as the mobilization-based characteristic value, X_k^{mob} , and referred to as "MP18" and "M19" in this report:

MP18:
$$X_k^{mob} = X_m \times (1 + \alpha_{5\%} \Gamma_I V_{TOT})$$
 for Normal distribution (6.16a)

MP19:
$$X_k^{mob} = X_m \times \exp\left(\alpha_{5\%} \times \sqrt{\ln\left[1 + \Gamma_I^2 V_{TOT}^2\right]}\right) / \sqrt{1 + \Gamma_I^2 V_{TOT}^2}$$
 for Lognormal distribution (6.16b)

where $\alpha_{5\%}$ is the 5% fractile of the minimum of n_s standard normal random variables, and it can be

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$$\alpha_{5\%} = \Phi^{-1} \left[1 - 0.95^{1/n_{s}} \right]$$
(6.17)

As indicated by Eq. (6.17), the $\alpha_{5\%}$ is reduced to -1.645 when $n_s = 1$. The $\alpha_{5\%}$ has been calibrated from RFEM for several geotechnical problems for a scenario of $V_X = 0.3$ and $\delta_X = 10 \times \delta_Z$, as shown in Table 3. The Γ_I in Eq. (6.16) is the variance reduction factor along a classical slip surface. It can be determined by Eqs. (6.8) and (6.9) with L = the length of the classical slip surface and δ_L = SOF along the classical slip surface, which can be approximately estimated as (Vanmarcke 1977):

$$\delta_L \approx \left[\left(\frac{\Delta x}{\Delta x + \Delta z_1 + \Delta z_2} \right) \times \frac{1}{\delta_X} + \left(\frac{\Delta z_1 + \Delta z_2}{\Delta x + \Delta z_1 + \Delta z_2} \right) \times \frac{1}{\delta_Z} \right]^{-1}$$
(6.18)

where Δz_1 , Δz_2 , and Δx are the length of segments approximating the classical slip surface, e.g., see Figure 6.2 for the footing foundation. Note that characteristic values derived from MP17-MP19 correspond to the 5% quantile of the geotechnical structure response while the characteristic values derived from MP1-MP16 refer to the 5% quantile of a single soil parameter. This makes a huge difference for multi-layered and multi-variate problems, as discussed later.

Problem	Type of slip curve	$a_{5\%}$ (for $V_X = 0.3$ and $\delta_X = 10 \times \delta_Z$		
Soil column subjected	Slip augus is lowly constrained	$a = -0.48 \times (5.71)^{-0.36} + 6.45$		
to axial loading	Sup curve is lowly constrained	$\alpha_{5\%} = -0.48 \times (\partial_L/L)^{-1.043}$		
Shallow foundation	Slip curve is intermediately	$a = 0.21 \times (8/1)^{-0.55} + 6.45$		
Basal heave stability	constrained	$\alpha_{5\%} = -0.21 \times (O_L/L)^{-1.045}$		
Retaining wall	Slip curve is highly constrained	$\alpha_{5\%} = -0.04 \times (\delta_L/L)^{-0.87} - 1.645$		
Friction pile	Slip any is fully constrained	a. – 1645		
(compressive loading)	Sup curve is runy constrained	$a_{5\%} = -1.045$		

Table 6.3. Equations for $\alpha_{5\%}$ for different problems (source: Tabarroki et al. 2020)



Figure 6.2. Approximation of the classical slip curve for a footing foundation (ϕ =0) (source: Tabarroki et al. 2020)

6.3.6 Characteristic value of the profile of geotechnical parameters over depth

Some geotechnical parameters may exhibit an obvious spatial trend over depth. Such a spatial trend was not considered by MP1-MP19 described previously. For example, if a linear spatial trend is observed, the profile of X over depth can be represented as (e.g., Frank 2004):

$$X = X_m + b\left(z - \overline{z}\right) \tag{6.19}$$

where \overline{z} is the depth of the gravity center of a number, *n*, of *X* measurements (i.e., $[x_1, x_2, ..., x_n]$) at different depths $z_1, z_2, ..., z_n$; X_m is the statistical mean of the *n* measurements; *b* is a regression coefficient and represents the slope of the linear trend line over depth. Then, for local and global failure scenarios, the characteristic values, $X_{k,z}$, of *X* at the depth *z* can be calculated using the following formulas (e.g., Frank 2004; Prästings et al. 2019):

MP20:
$$X_{k,z} = \left[X_m + b\left(z - \overline{z}\right)\right] - t_{95,n-2} \times s_{x|z}$$
 for local failure scenarios (6.20a)

MP21:
$$X_{k,z} = [X_m + b(z - \overline{z})] - t_{95,n-2} \times s_{\overline{x}|z}$$
 for global failure scenarios (6.20b)

where $t_{95,n-2}$ is a Student's-*t* factor evaluated for a 95% confidence level and *n*-2 degrees of freedom; $s_{x|z}$ and $s_{\overline{x}|z}$ are respective standard deviations of *X* and its mean value at a depth of *z*, and they are given by:

$$s_{x|z} = \sqrt{\frac{1}{n-2} \left(1 + \frac{1}{n} + \frac{\left(z - \overline{z}\right)^2}{\sum_{i=1}^n \left(z_i - \overline{z}\right)^2} \right)^2} \sum_{i=1}^n \left[\left(x_i - X_m\right) - b\left(z_i - \overline{z}\right) \right]^2$$
(6.21a)

$$s_{\bar{x}|z} = \sqrt{\frac{1}{n-2} \left(\frac{1}{n} + \frac{(z-\bar{z})^2}{\sum_{i=1}^n (z_i - \bar{z})^2} \right)^2} \sum_{i=1}^n \left[(x_i - X_m) - b(z_i - \bar{z}) \right]^2$$
(6.21b)

Determining the characteristic value of the profile of geotechnical parameters over depth can be more challenging if a non-linear spatial trend is exhibited by test data though some methods have been suggested to assess such trends (e.g., Müller et al 2016; Spross and Larsson 2019). This challenge is more profound when only sparse data over depth are obtained from site investigation. Recently, Wang and Zhao (2017) and Zhao et al. (2018) proposed a Bayesian compressive sensing (BCS) approach for probabilistic interpolation of the profile of geotechnical parameters over depth from sparse data. Ching and Phoon (2020) proposed a Bayesian approach that can handle MUSIC-X site investigation data, where M stands for "multivariate", U for "uncertain and unique", S for "sparse", I for "incomplete", C for "possibly corrupted", and X for "spatially varying". This approach considers both the spatial correlation (correlation among different locations) and cross correlation (cross correlation among different soil properties) when interpolating the profile of geotechnical parameters. These approaches not only provide the expected (or mean) profile of the geotechnical parameter concerned but also give the confidence interval of the estimated profile to quantify its associated uncertainty. The lower bound of the confidence interval can be taken as a quantile profile of the geotechnical parameter. For example, the lower bound of the 90% confidence interval of the estimated profile from BCS corresponds to the 5% quantile profile, which can be taken as the characteristic value over depth. These BCS-based and MUSIC-X methods for determining the profile of the characteristic value are referred to as "MP22" and "MP23" in this report. Detailed algorithms and implementation of BCS and MUSIC-X can be referred to Wang and Zhao (2017), Zhao et al. (2018), Zhao and Wang (2020), Zhao et al. (2020), and Ching and Phoon (2020).

6.3.7 Characteristic value of multiple correlated geotechnical parameters

Geotechnical designs may involve multiple geotechnical parameters as input. A straightforward way to determine multivariate characteristic values is to specify the same fractile (e.g., 5% fractile) for each geotechnical parameter to obtain a vector (i.e., X_k) of characteristic values of multiple geotechnical parameters (e.g., Tang et al. 2018; Ching et al. 2020). This method, however, does not consider the correlation among geotechnical parameters. Alternatively, multivariate characteristic values can be back-calculated from the cumulative distribution function (CDF) of the performance function concerned in design, which gives reliability-based multivariate characteristic values (e.g., Hicks et al. 2019; Ching et al. 2020). Determining reliability-based multivariate characteristic values often requires probabilistic analyses that may not be familiar to geotechnical practitioners. To address this issue, Ching et al. (2020) proposed a practical approach to determine reliability-based multivariate characteristic values based on the "effective random dimension" (ERD). The ERD characterizes the degree of redundancy of the performance function, and it represents the effective number of independent standard normal random variables that affect the limit state (Ching et al. 2015). Detailed calculations of ERD for a given performance function are referred to Ching et al. (2015, 2020). Based on the ERD, the required η for selecting multivariate characteristic values for the 5% fractile performance is determined as follows:

$$\eta = \Phi \left[-1.645 / \sqrt{ERD} \right] \tag{6.22}$$

Varkey et al. (2020) further extended Eq. (22) to account for the spatial averaging effect over the influence zone:

$$\eta = \Phi \left[-1.645 \times \Gamma_I / \sqrt{ERD} \right] \tag{6.23}$$

where Γ_I^2 is the variance reduction function quantifying the variance reduction due to spatial averaging over the influence zone. Once η is computed, the characteristic value for each of the multivariate inputs should be taken as its η fractile, rather than the 5% fractile. This method of selecting multivariate characteristic values is referred to as "MP24" in this report.

6.3.8 Characteristic value as an average with qualitative adjustment to factors F1-F6

Rather than trying to address the effect of factors F1-F6 by calculation, the Swedish Implementation Commission of European Standards in Geotechnics (IEG) introduced a practice where a best-estimate value is adjusted to the relevant features of F1-F6 through a conversion factor η . This practice utilizes an opening in Eurocode EN 1990, principle 4.2(4)P, which states that "a conversion factor shall be applied [to the characteristic value] where it is necessary to convert the test results into values which can be assumed to represent the behaviour of the material or product in the structure or the ground".

For convenience, the Swedish practice redefines the characteristic value so that the conversion factor becomes an integrated part of it:

MP25:
$$X_k = \eta X_{chosen}$$
 (6.24)

where X_{chosen} is either a subjective, best-estimate (i.e. non-cautious) mean value, or a calculated mean value, X_m . The X_{chosen} may be a weighted mean value with respect to the accuracy of the used investigation method, e.g. through a multivariate approach (Ching et al. 2010; Prästings et al. 2017). All uncertainty in the characteristic value is consequently assigned to η . The Swedish implementation guidelines to Eurocode 7 (IEG, 2008) defines η as a product of eight sub-factors: $\eta = \eta_1 \eta_2 \eta_3 \eta_4 \eta_5 \eta_6 \eta_7 \eta_8$. Table 6.4 shows their respective purposes. Each sub-factor η_i has been calibrated by IEG for a large number of combinations of geotechnical structure types and soil types, to address the features of F2-F6 (while F1 is addressed by X_{chosen}). The number of possible combinations are extensive and not presented here. Though, to give a brief example, sub-factor η_3 is either 0.9, 0.95, 1.0, 1.05, or 1.1, when evaluating undrained shear strength in a slope stability problem; the selected value depends both on how many different investigation methods that were used and on the judged variability of the results, as detailed in a description for each value.

Typically, the total η would end up between 0.8 and 1.2 for most combinations of geotechnical structures and soil types. Thus, the X_k can notably be larger than X_m , which may seem counterintuitive at first, considering the often expressed need for cautiousness in determining characteristic values. The reason is that the later applied fixed partial factor, γ_M , provides a substantial safety margin when determining the design value ($X_d = X_k / \gamma_M$) and such a large safety margin was not found to be needed for all situations, i.e. in case of very favorable conditions with high-quality geotechnical investigations. By adopting the conversion factor η , IEG managed to

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Table 6.4. Definition of conversion sub-factors η_i that modifies X_k in the Swedish implementation guidelines to Eurocode 7, and their connection to factors *F1-F6*

η_i	Definition	F1-F6
η_1	The inherent variability of the material properties	F2
η_2	Number of independent measurement points	F3
η_3	Uncertainty related to the assessment of soil properties	F2, F3, F4
η_4	The location of measurement points in relation to the structure	F3
η_5	The extent of the ground zone governing the behaviour of the	F5
	geotechnical structure at the limit state being considered	
η_6	The ability of the geotechnical structure to transfer loads from	<i>F6</i>
	weak to strong zones in the ground	
η_7	Type of failure mechanism (i.e. ductile or brittle failure)	(-)
η_8	The sensitivity of the material design property on the limit state	(-)

6.4 Methods of determining characteristic values of geotechnical resistance

When resistance factor design methods are applied, the characteristic value, R_k , of geotechnical resistance is needed. It can be estimated from characteristic values of geotechnical parameters using a design calculation model M, which can be written as (Bond and Harries 2008):

MR1:
$$R_k = M(\underline{X}_k)$$
 (6.25)

Fenton et al. (2016) pointed out that the variability of geotechnical resistance is generally less than the variability of geotechnical parameters at a point level due to spatial averaging, and suggested that the R_k can be evaluated as:

MR2:
$$R_k = \mu_R / k_R = \mu_R (1 - 1.645 V_R)$$
 (6.26)

where k_R is the resistance bias factor; μ_R and V_R are the mean value and coefficient of variation of geotechnical resistance, respectively. As indicated by Eq. (6.26), the resistance bias factor k_R (i.e., μ_R/R_k) is equal to $1/(1-1.645V_R)$. Assuming $V_R = 0.15$, the k_R is calculated as 1.33 for Design Approach 2 in Eurocode 7 (Fenton et al. 2016). The same k_R was also assumed for Australia by Fenton et al. (2016) while the $k_R = 1.1$ was assumed in North American design codes (e.g., CHBDC, NBCC, and AASHTO LRFD BDS).

For pile designs, Eurocode 7 also provides correlation factors to determine characteristic values of pile resistance based on static pile load tests, ground test results, and dynamic impact tests.

MR3:
$$R_{\rm k} = {\rm Min}\left\{\frac{R_{\rm mean}}{\xi_i}; \frac{R_{\rm min}}{\xi_j}\right\}$$
 $i = 1, 3, 5 \text{ and } j = 2, 4, 6$ (6.27)

where R_{mean} and R_{min} are the mean and minimum values of the pile resistance measured from load tests or calculated from ground test results; ξ_1 and ξ_2 are correlation factors to derive characteristic values from static pile load tests; ξ_3 and ξ_4 are correlation factors to derive characteristic values from ground test results; ξ_5 and ξ_6 are correlation factors to derive characteristic values from dynamic impact tests. The values of ξ_1 - ξ_6 are provided in Appendix A.3.3.3 of Eurocode 7 (CEN 2004), which are shown in Table 6.5-6.7. Detailed discussions on these correlation factors can be found in Bauduin (2002) and Orr (2017).

Table 6.5. Correlation factors ξ to derive characteristic values from static pile load tests (After CEN 2004)

2007)									
ξ for $n =$	1	2	3	4	≥ 5				
ξ_1	1.40	1.30	1.20	1.10	1.00				
ξ_2	1.40	1.20	1.05	1.00	1.00				

Table 6.6. Correlation factors ξ to derive characteristic values from ground test results (After CEN 2004)

ξ for $n =$	1	2	3	4	5	7	10
ξ3	1.40	1.35	1.33	1.31	1.29	1.27	1.25
ξ_4	1.40	1.27	1.23	1.20	1.15	1.12	1.08

Table 6.7. Correlation factors ξ to derive characteristic values from dynamic impact tests (After CEN 2004)

		СП	2001)		
ξ for $n =$	≥ 2	≥5	≥10	≥15	≥ 20
ξ5	1.60	1.5	1.45	1.42	1.40
ξ_6	1.60	1.35	1.3	1.25	1.25

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Methods	F1	F2	<i>F3</i>	F4	<i>F</i> 5	F6
MP1	\bigcirc	\bullet	\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP2	\bigcirc	\bullet	\bigcirc	\bullet	\bigcirc	\bigcirc
MP3	\bigcirc	\bullet	\bigcirc	\bullet	\bigcirc	\bigcirc
MP4	\bigcirc		\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP5	\bigcirc	\bullet	\bigcirc		\bigcirc	\bigcirc
MP6	\bigcirc	\bullet	\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP7	\bigcirc	\bullet	\bigcirc		\bigcirc	\bigcirc
MP8	\bigcirc		\bigcirc		\bullet	\bigcirc
MP9	\bigcirc	\bullet	\bigcirc	\bullet	\bullet	\bigcirc
MP10	\bigcirc	\bullet	\bigcirc	\bigcirc	\bullet	\bigcirc
MP11	\bigcirc	\bullet	\bigcirc	\bigcirc	\bullet	\bigcirc
MP12	\bigcirc	\bullet	\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP13	\bigcirc	\bullet	\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP14	\bigcirc	\bullet	\bigcirc	\bigcirc	\bigcirc	\bigcirc
MP15	\bigcirc	\bullet	\bigcirc			\bigcirc
MP16	\bigcirc	\bullet	\bigcirc			\bigcirc
MP17	\bigcirc	\bullet	\bigcirc	\bigcirc		\bullet
MP18	\bigcirc	\bullet	\bigcirc	\bigcirc		\bigcirc
MP19	\bigcirc	\bullet	\bigcirc			\bigcirc
MP20	\bigcirc	\bullet	\bullet	\bigcirc	\bigcirc	\bigcirc
MP21	\bigcirc	\bullet	\bullet		\bigcirc	\bigcirc
MP22	\bullet	\bullet	\bullet	\bigcirc	\bigcirc	\bigcirc
MP23	\bullet	\bullet	\bullet		\bigcirc	\bigcirc
MP24	\bigcirc	\bullet	\bullet	\bigcirc		\bigcirc
MP25	\bigcirc	\bigcirc	\bigcirc	\bigcirc	\bigcirc	

 Table 6.8. Overview of factors (F1-F6) addressed by different methods (MP1-MP25) for determining geotechnical characteristic parameters

Note: •: Addressed; • Partially addressed; •: Not addressed

6.5 Summary

This report provides an overview of statistical methods of determining characteristic values of geotechnical parameters and resistance. Definitions of characteristic values of geotechnical parameters and resistance in geotechnical design codes (including Eurocodes, CHBDC in Canada, ASSHTO LRFD BDS in USA, Australia Standard, and CIGE in China) and the literature were summarized in Tables 6.1 and 6.2. These definitions of geotechnical characteristic values are often worded to avoid being too prescriptive. This provides flexibility to geotechnical engineers to select characteristic values of geotechnical parameters and resistance for a specific project by exercising their engineering judgment and experience in a subjective way.

Statistical methods (including frequentist and Bayesian approaches) allow determining characteristic values of geotechnical parameters (e.g., MP1-MP25) and resistance (e.g., MR1-MR3) in a quantifiable way. The MP1-MP19 were developed based on distributions of different random variables (including basic random variable X, statistical mean X_m , spatial average X_A , and effective or mobilized property X_E). The MP20-MP23 can be used to determine the profile of characteristic geotechnical parameters over depth, and the MP24 gives the characteristic values of multiple parameters with consideration of cross-correlation among them. Characteristic values derived from MP1-MP16 and MP20-MP22 refer to the 5% quantile of a single soil parameter or its profile over depth, and MP23 (MUSIC-X method) is able to give the profiles of multivariate characteristic values as the same quantile of multiple soil parameters. In contrast, the characteristic values calculated by MP17-MP19 and MP24 correspond to the 5% quantile of the geotechnical structure response. The MP1-MP24 were developed based on different assumptions considering different uncertainties in estimated geotechnical parameters, and some factors among *F1-F6* mentioned in Eurocode 7, Clause 2.4.5.2(4)P were taken into account, as summarized in Table 6.8.

The MP17 and MP25 were found to be the only methods that are able to address all the six factors. For MP17 this is thanks to the prominent flexibility of uncertainty and deterministic modelling provide by RFEM. Nevertheless, it shall be pointed out that, in practice, whether characteristic values derived from MP17 account for all the six factors is a matter of detailed modelling in implementation. MP25 addresses the factors F1-F6 in a more qualitative way through a combination of many pre-calibrated conversion factors, which are provided to the engineer in a large number of tables. While less mathematically rigorous than many of the other methods, MP25 provides a straightforward way for the practicing engineer to account explicitly for all aspects F1-F6.

As for characteristic geotechnical resistance, it can be calculated either from characteristic values of geotechnical parameters using a design calculation model (i.e., MR1) or by applying the resistance bias factor (i.e., MR2). Eurocode 7 also provides correlation factors to determine characteristic pile resistances based on static pile load tests, ground test results, and dynamic impact tests (i.e., MR3).

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7. Numerical evidences for worst-case scale of fluctuation

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7.1 Introduction

The worst-case scale of fluctuation (worst-case SOF) is a phenomenon observed in the literature (e.g., Fenton and Griffiths 2003; Jaksa et al. 2005; Fenton et al. 2005; Breysse et al. 2005; Griffiths et al. 2006; Soubra et al. 2008; Vessia et al. 2009; Ching and Phoon 2013; Ahmed and Soubra 2014; Hu and Ching 2015; Stuedlein and Bong 2017) where more complex non-classical mechanisms can occur when the SOF of the soil spatial variability (usually modeled by a random field) is comparable to some multiple of the characteristic length of the geotechnical structure (e.g., width of a footing, distance between shallow pads, height of a retaining wall). The complex behavior typically manifests itself most clearly when the mean response from random field realizations is compared with the nominal response produced by a soil mass taking mean properties everywhere. At the worst-case SOF, the discrepancy between the mean response and the nominal response is the largest. The existence of a worst-case SOF indicates that adopting the nominal response is unconservative. The concept of the worst-case SOF, the use of the worst-case SOF may ensure a conservative design.

7.2 Summary Table

Table 7.1 summarizes past studies that report worst-case SOFs. The characteristic length in the fifth column is mostly chosen by the current report rather than by the past studies. Right now, the characteristic length is mostly chosen to be related to the dimension of the geotechnical structure.

7.3 Key observations

- 1. In terms of problem types, only five problem types have been investigated by the past studies in Table 7.1, including footing (43% of the studies), retaining wall (14%), soil column (18%), basal heave (7%), slope (11%), and pile (7%). In terms of limit states, 75% of the studies are for the ultimate limit state (ULS) (strength and capacity), whereas 25% are for the serviceability limit state (SLS) (settlement and deformation).
- 2. All studies adopted numerical models to investigate worst-case SOFs. Among them, 72% of the studies adopted random finite element analyses (RFEA), 7% adopted random finite difference analyses (RFDA), 11% adopted random limit equilibrium methods (RLEM), and 11% adopted numerical models calibrated by RFEA/FFDA/RLEM. None of the studies in the table investigated worst-case SOFs based on real-world field tests or model tests.
- 3. In terms of problem dimensions, 7% of the studies investigated 1D problems, 75% investigated 2D problems, and 18% investigated 3D problems.

- 4. The definitions for worst-case SOFs are not uniform. Most of them are defined as the SOF that produces the minimal mean strength/capacity/FS, maximal under-design probability, or maximal mean demand (e.g., mean settlement, mean moment, mean lateral force, etc.).
- 5. For many cases in the table, the worst-case SOFs are found to be around the characteristic length (worst-case SOF ≈ characteristic length), but there are also cases where the worst-case SOF is far from the characteristic length.
- 6. For the cases focusing on ULS, the ratio (worst-case mean response)/(nominal response) varies from 0.5 to 1.0, where "nominal response" refers to the response produced by a homogenous analysis with soil parameter fixed at the mean value of the random field. This ratio seems to depend on the following three factors: (a) COV of random field; (b) problem type; (c) isotropy/anisotropy. Figure 7.1 shows how this ratio varies with COV. Figure 1a shows that the ratio is larger for anisotropic cases ($\delta_h > \delta_v$) and smaller for isotropic cases ($\delta_h = \delta_v$). Figure 7.1b shows that the ratio is the largest for retaining wall and smallest for soil column.
- 7. For compressive strength of a soil column, some studies investigated 3D problems. Figure 7.1c shows the ratios (worst-case mean response)/(nominal response) for both 2D and 3D cases. It seems that this ratio is not significantly affected by the problem dimension.



Figure 7.1. Variation of (worst-case mean response)/(nominal response) with respect to COV: (a) isotropic v.s. anisotropic cases; (b) different problem types; (c) 2D v.s. 3D soil column

7.4 References

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Reference	Response & problem type	Worst-case definition	Simulation method	Characteristic length	Random field characteristics: 1D/2D/3D Isotropic/anisotropic COV	Worst-case SOF	(Worst-case mean response)/ (nominal response)
Jaksa et al. (2005)	Settlement of a nine-pad footing system	Under-design probability is maximal	RFEA	Footing spacing (S)	3D random field of E $\delta_{h}/\delta_{v} = 1$ and 2 COV = 0.1 and 0.5	1×S	
Fenton and Griffiths (2005)	Differential settlement between two footings	Standard deviation of different settlement is maximal	RFEA	Footing spacing (S)	3D random field of E $\delta_h/\delta_v = 1$ COV = 0.1 to 4	1×S	
Fenton and Griffiths (2003)	Bearing-capacity factor of a strip footing on a c-\u03c6 soil	Mean of log bearing-capacit y factor is minimal	RFEA	Footing width (B)	2D random fields of c and ϕ $\delta_h/\delta_v = 1$ COV = 0.1 to 5	1 to 5×B	0.88 for COV = 0.1 0.81 for COV = 0.2 0.73 for COV = 0.5 0.62 for COV = 1 0.51 for COV = 2 0.45 for COV = 5
Soubra et al. (2008)	Bearing capacity under punching failure mode for a strip footing	Mean bearing capacity is minimal	RFDA	Footing width (B)	2D random fields of c and ϕ $\delta_{h}/\delta_{v} = 0.25$ to 25 COV _c = 0.2 to 0.4 COV_{\phi} = 0.1 to 0.15	1×B for isotropic 5×B for anisotropic	$\begin{array}{l} 0.93 \mbox{ for } COV_c = 0.2 \mbox{ \& } COV_{\phi} = 0.1 \\ 0.90 \mbox{ for } COV_c = 0.4 \mbox{ \& } COV_{\phi} = 0.1 \\ 0.89 \mbox{ for } COV_c = 0.2 \mbox{ \& } COV_{\phi} = 0.15 \end{array}$
Fenton et al. (2005)	Active lateral force for a retaining wall	Under-design probability is maximal	RFEA	Wall height (H)	2D random fields of ϕ and γ $\delta_h/\delta_v = 1 \\ COV = 0.02 \text{ and } 0.5$	0.5 to 1×H	
		Mean footing rotation is maximal			1D random field of soil subgrade modulus k	0.5×S	
Breysse et al. (2005)	Settlement of a footing system	Mean sewer pipe bending moment is maximal	Numerical model + MCS	Footing spacing (S) & Pipe length		1 to 2×L	
		Mean different settlement between footings is		(L)		Very complicated	

 Table 7.1. Summary of worst-case SOFs in the literature

		movimol					
Griffiths et al. (2006)	Bearing capacity of a strip footing on $\phi=0$ soil	Mean bearing capacity is minimal	RFEA	Footing width (B)	$\begin{array}{l} 2D \text{ isotropic random field of } s_u\\ \delta_h/\delta_v=1\\ COV=0.25 \text{ to } 8 \end{array}$	0.5 to 1×B	0.58 for COV = 1
Vessia et al. (2009)	Bearing capacity of a strip footing on c-\u00f6 soil	Mean bearing capacity is minimal	RFEA	Footing width (B)	2D random fields of c and ϕ $\delta_h/\delta_v = 0.5$ to 25 $COV_c = 0.56$ & $COV_{\phi} = 0.24$	1×B for isotropic 0.3 to 0.5×B for anisotropic	$\begin{array}{l} 0.80 \mbox{ for } \delta_h/B = 1 \\ 0.82 \mbox{ for } \delta_h/B = 5 \\ 0.85 \mbox{ for } \delta_h/B = 10 \\ 0.87 \mbox{ for } \delta_h/B = 30 \\ 0.88 \mbox{ for } \delta_h/B = 50 \end{array}$
Ching and Phoon (2013)	Compressive strength of a undrained soil column	Mean strength is minimal	RFEA	Column width (W)	$\begin{array}{l} \text{2D random field of } s_u \\ \delta_h / \delta_v = 1 \ \& \ \infty \\ \text{COV} = 0.2 \end{array}$	1×W	$\begin{array}{l} 0.8 \mbox{ for } \delta_h / \delta_v = 1 \\ 0.82 \mbox{ for } \delta_h / \delta_v = \infty \end{array}$
Ahmed and Soubra (2014)	Differential settlement between strip footings	Under-design probability is maximal	RFDA	Footing spacing (S)	$\begin{array}{l} \text{2D random field of E} \\ \delta_h/\delta_v = 1 \text{ to } 30 \\ \text{COV} = 0.15 \end{array}$	1×S	
Hu and Ching (2015)	Active lateral force for a retaining wall in clay	Mean active lateral force is maximal	RLEM	Wall height (H)	2D random field of $s_u \\ \delta_h / \delta_v = 1 \\ COV = 0.1 \text{ to } 0.3$	0.1 to 0.2×H	1.24 for COV = 0.3
Stuedlein and Bong (2017)	Differential settlement of footings	Under-design probability is maximal	Numerical model + MCS	Footing spacing (S)	2D conditional random field based on CPT data	1×S	
Ali et al. (2014)	Risk of an infinite slope	Risk of rainfall induced slope failure is maximal	RLEM	Slope thickness (H)	1D random field of hydraulic conductivity k COV = 1	1×H	
Pan et al. (2018)	Compressive strength of a cement-treated clay column	Mean strength is minimal	RFEA	Column diameter (D)	3D random field of s_u $\delta_h/\delta_v = 1$ COV = 0.2 to 0.4	1×D	0.84 for COV = 0.2 0.74 for COV = 0.3 0.66 for COV = 0.4
Pula et al. (2017)	Bearing capacity of a strip footing on c-¢ soil	Under-design probability is maximal	RFEA	Footing width (B)	Scenario 1: 2D random fields of c $\delta_h/\delta_v = 1$ COV _c = 0.1 to 4 Scenario 2: 2D random fields	Scenario 1: 8×B Scenario 2: local maximum effect not observed	

					$eq:started_st$		
Javankhoshd el et al. (2017)	FS of a cohesive slope	Probability of failure is maximal	RFEA/RLE M	Slope height (H)	2D random field of $s_u = \delta_h / \delta_v$ varies COV = 0.5		
Luo and Bathurst (2017)	Bearing-capacity factor of a strip footing on a cohesive slope	Mean bearing-capacit y factor is minimal	RFEA	Footing width (B)	$\begin{array}{l} \text{2D random field of } s_u \\ \delta_h / \delta_v = 1 \text{ to } 20 \\ \text{COV} = 0.5 \end{array}$	0.25 to 1×B	Very complicated, depending on several factors
Allahverdizade h et al. (2016)	Compressive strength of a square block	Mean compressive strength is minimal	RFEA	Block width (B)	2D random fields of c and ϕ $\delta_h/\delta_v = 1$ $COV_c = COV_{\phi} = 0.2 \text{ to } 0.5$	0.1 to 1×B	0.86 for COV = 0.2 0.80 for COV = 0.3 0.67 for COV = 0.5
Griffiths et al. (2008)	Passive lateral force for a retaining wall	Under-design probability is maximal	RFEA	Wall height (H)	2D random fields of c and ϕ $\delta_h/\delta_v = 1$ COV = 0.02 to 0.5	0.1 to 0.5×H	
Tabarroki et al. (2013)	FS of a cohesive slope	Mean FS is minimal	RFEA/RLE M	Slope height (H)	$\begin{array}{l} \text{2D random field of } s_u \\ \delta_h / \delta_v = 1 \\ \text{COV} = 0.1 \text{ to } 0.5 \end{array}$	0.1~1×H	0.97 for COV = 0.1 0.91 for COV = 0.3 0.80 for COV = 0.5
Namikawa and Koseki (2013)	Compressive strength of a cement-treated column	Mean strength is minimal	RFEA	Column diameter (D)	3D random field of q_u $\delta_h/\delta_v = 1$ COV = 0.2 to 0.4	0.5×D	0.88 for COV = 0.2 0.81 for COV = 0.3 0.73 for COV = 0.4
Ching et al. (2017)	FS for basal heave of an excavation in clay	Mean FS is minimal	RFEA	Wall penetration depth (H _p)	2D random field of s_u/σ'_v $\delta_b/\delta_v = 1$ to 100 COV = 0.1 to 0.5	0.1~0.4×Hp	$\begin{array}{l} 0.98 \mbox{ for } \delta_h/\delta_v = 1 \mbox{ \& COV } = 0.1 \\ 0.9 \mbox{ for } \delta_h/\delta_v = 1 \mbox{ \& COV } = 0.3 \\ 0.81 \mbox{ for } \delta_h/\delta_v = 1 \mbox{ \& COV } = 0.5 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 10 \mbox{ & COV } = 0.1 \\ 0.93 \mbox{ for } \delta_h/\delta_v = 10 \mbox{ & COV } = 0.3 \\ 0.86 \mbox{ for } \delta_h/\delta_v = 10 \mbox{ & COV } = 0.5 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 30 \mbox{ & COV } = 0.1 \\ 0.94 \mbox{ for } \delta_h/\delta_v = 30 \mbox{ & COV } = 0.3 \\ 0.89 \mbox{ for } \delta_h/\delta_v = 30 \mbox{ & COV } = 0.5 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \mbox{ & COV } = 0.1 \\ \end{array}$

							$\begin{array}{c} 0.96 \mbox{ for } \delta_h / \delta_v = 100 \mbox{ & COV} = 0.3 \\ 0.9 \mbox{ for } \delta_h / \delta_v = 100 \mbox{ & COV} = 0.5 \end{array}$
Naghibi et al. (2016)	Differential settlement of a two-pile system	Mean differential settlement is maximal	Semi-theore tical model validated by RFEA	Pile spacing (S)	2D random field of E $\delta_h/\delta_v = 1$ COV = 0.1 to 0.5	1×S	
Leung and Lo (2018)	Differential settlement of a pile group	Standard deviation of differential settlement is maximal	RFEA	Width of pile group (B) Length of pile group (L)	3D random field of E $\delta_h/B = 0.5, 1 \text{ and } \infty$ $\delta_v/L = 0.15 \text{ to } 1$ COV = 0.25 and 0.5	0.5 to 1×B	
Tabarroki (2020)	Mobilized shear strength of a soil column under compression	Mean mobilized shear strength is minimal	RFEA	Column width (W)	Scenario A: 2D random field of s _u Scenario B: 2D random field of s _u / σ'_v Scenario C: 2D random field of tan ϕ' $\delta_h/\delta_v = 1$ to 100 For A & B: COV = 0.1 to 0.5 For C: COV = 0.05 to 0.2	0.5×W	$\begin{array}{c} Scenario A:\\ 0.93 \mbox{ for } \delta_h/\delta_v = 1 \ \& \ COV = 0.1 \\ 0.78 \mbox{ for } \delta_h/\delta_v = 1 \ \& \ COV = 0.3 \\ 0.65 \mbox{ for } \delta_h/\delta_v = 10 \ \& \ COV = 0.5 \\ 0.93 \mbox{ for } \delta_h/\delta_v = 10 \ \& \ COV = 0.1 \\ 0.78 \mbox{ for } \delta_h/\delta_v = 10 \ \& \ COV = 0.3 \\ 0.64 \mbox{ for } \delta_h/\delta_v = 10 \ \& \ COV = 0.1 \\ 0.78 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.1 \\ 0.78 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.3 \\ 0.65 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.3 \\ 0.65 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.79 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.79 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.3 \\ 0.65 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.3 \\ 0.65 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.87 \mbox{ for } \delta_h/\delta_v = 1 \ \& \ COV = 0.2 \\ 0.97 \mbox{ for } \delta_h/\delta_v = 10 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.2 \\ 0.97 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.2 \\ 0.95 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.2 \\ 0.95 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 30 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.88 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.2 \\ 0.98 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0.1 \\ 0.85 \mbox{ for } \delta_h/\delta_v = 100 \ \& \ COV = 0$
						$0.89 \text{ for } \delta_h / \delta_v = 100 \& \text{ COV} = 0.2$	
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						Scenario A:	
						0.98 for $\delta_h/\delta_v = 1$ & COV = 0.1	
						0.9 for $\delta_h/\delta_v = 1$ & COV = 0.3	
						0.81 for $\delta_h/\delta_v = 1$ & COV = 0.5	
						0.99 for $\delta_h/\delta_v = 10$ & COV = 0.1	
						0.95 for $\delta_h / \delta_v = 10$ & COV = 0.3	
						0.88 for $\delta_h/\delta_v = 10$ & COV = 0.5	
					1 for $\delta_h/\delta_v = 30$ & COV = 0.1		
						0.98 for $\delta_h/\delta_v = 30$ & COV = 0.3	
						0.94 for $\delta_h / \delta_v = 30$ & COV = 0.5	
						1 for $\delta_h/\delta_v = 100$ & COV = 0.1	
						0.99 for $\delta_h/\delta_v = 100$ & COV = 0.3	
						0.97 for $\delta_h/\delta_v = 100$ & COV = 0.5	
						Scenario B:	
						$0.99 \text{ for } \delta_{\text{b}}/\delta_{\text{v}} = 1 \& \text{COV} = 0.1$	
	Mobilized shear					0.92 for $\delta_{\rm h}/\delta_{\rm v} = 1$ & COV = 0.3	
	strength of a		Wall height	0.05×H	$0.83 \text{ for } \delta_{\text{b}}/\delta_{\text{v}} = 1 \& \text{COV} = 0.5$		
	retaining wall			(H)		1 for $\delta_{\rm b}/\delta_{\rm v} = 10$ & COV = 0.1	
	C					0.97 for $\delta_{\rm h}/\delta_{\rm v} = 10$ & COV = 0.3	
						0.92 for $\delta_{\rm h}/\delta_{\rm v} = 10$ & COV = 0.5	
						1 for $\delta_{\rm h}/\delta_{\rm v} = 30$ & COV = 0.1	
						0.99 for $\delta_{\rm h}/\delta_{\rm v} = 30$ & COV = 0.3	
					0.96 for $\delta_{\rm h}/\delta_{\rm v} = 30$ & COV = 0.5		
					1 for $\delta_{\rm h}/\delta_{\rm v} = 100$ & COV = 0.1		
						0.99 for $\delta_h / \delta_v = 100 \& \text{COV} = 0.3$	
						0.97 for $\delta_h\!/\delta_v = 100$ & COV = 0.5	
						Scenario C	
						1 for $\delta_{\rm b}/\delta_{\rm c} = 1 \& \rm COV = 0.05$	
						$0.98 \text{ for } \delta_{\text{b}}/\delta_{\text{v}} = 1 \& \text{COV} = 0.03$	
						$0.95 \text{ for } \delta_{\text{h}}/\delta_{\text{v}} = 1 \& \text{COV} = 0.1$	
						$1 \text{ for } \delta_{\rm b}/\delta_{\rm v} = 10 \& \text{COV} = 0.2$	
						$0.99 \text{ for } \delta_{\text{b}}/\delta_{\text{v}} = 10 \& \text{COV} = 0.1$	
						$0.97 \text{ for } \delta_{\text{b}}/\delta_{\text{v}} = 10 \& \text{COV} = 0.2$	

							1 for $\delta_h / \delta_v = 30$ & COV = 0.05
							0.99 for $\delta_h / \delta_v = 30$ & COV = 0.1
							0.98 for $\delta_h / \delta_v = 30$ & COV = 0.2
							1 for $\delta_h / \delta_v = 100$ & COV = 0.05
							0.99 for $\delta_h/\delta_v = 100$ & COV = 0.1
							0.97 for $\delta_h/\delta_v = 100$ & COV = 0.2
							Scenario A:
							0.98 for $\delta_h / \delta_v = 1$ & COV = 0.1
							0.9 for $\delta_h/\delta_v = 1$ & COV = 0.3
							0.8 for $\delta_h/\delta_v = 1$ & COV = 0.5
							0.98 for $\delta_h/\delta_v = 10$ & COV = 0.1
							0.9 for $\delta_h / \delta_v = 10$ & COV = 0.3
							0.81 for $\delta_h/\delta_v = 10$ & COV = 0.5
							0.98 for $\delta_h/\delta_v = 30$ & COV = 0.1
						0.9 for $\delta_h/\delta_v = 30$ & COV = 0.3	
						0.8 for $\delta_h/\delta_v = 30$ & COV = 0.5	
						0.97 for $\delta_h/\delta_v = 100$ & COV = 0.1	
						0.88 for $\delta_h/\delta_v = 100$ & COV = 0.3	
						0.78 for $\delta_h / \delta_v = 100$ & COV = 0.5	
	Mobilized shear						
	strength of a strip footing		Footing width		0.1 to 0.5×B	Scenario B:	
				(B)			$0.98 \text{ for } \delta_{\rm h}/\delta_{\rm v} = 1 \& \text{COV} = 0.1$
						0.9 for $\delta_h/\delta_v = 1$ & COV = 0.3	
						0.8 for $\delta_h/\delta_v = 1$ & COV = 0.5	
						$0.98 \text{ for } \delta_{\text{h}}/\delta_{\text{v}} = 10 \& \text{COV} = 0.1$	
						$0.9 \text{ for } \delta_{\rm h}/\delta_{\rm v} = 10 \& \text{COV} = 0.3$	
						$0.8 \text{ for } \delta_{\rm h}/\delta_{\rm v} = 10 \& \text{COV} = 0.5$	
						$0.98 \text{ for } \delta_{\text{h}}/\delta_{\text{v}} = 30 \& \text{COV} = 0.1$	
						0.9 for $\partial_h / \partial_v = 30$ & COV = 0.3	
						0.8 for $\partial_h / \partial_v = 30$ & COV = 0.5	
						$0.99 \text{ for } \delta_h / \delta_v = 100 \& \text{COV} = 0.1$	
						0.91 for $\partial_h / \partial_v = 100 \& COV = 0.3$	
							$0.81 \text{ for } \delta_{h}/\delta_{v} = 100 \& \text{ COV} = 0.5$
							Scenario C:
							1 for 8./8 - 1.% COV = 0.05
							$11010_{\rm h}/0_{\rm V} - 1 \propto COV - 0.03$

				$0.98 \text{ for } \delta_h / \delta_v = 1 \& \text{COV} = 0.1$
				0.95 for $\delta_h / \delta_v = 1$ & COV = 0.2
				1 for $\delta_{\rm h}/\delta_{\rm v} = 10$ & COV = 0.05
				0.98 for $\delta_h / \delta_v = 10$ & COV = 0.1
				0.94 for $\delta_h/\delta_v = 10$ & COV = 0.2
				1 for $\delta_{\rm h}/\delta_{\rm v} = 30$ & COV = 0.05
				0.98 for $\delta_h/\delta_v = 30$ & COV = 0.1
				0.94 for $\delta_h/\delta_v = 30$ & COV = 0.2
				1 for $\delta_h/\delta_v = 100 \& COV = 0.05$
				0.98 for $\delta_h/\delta_v = 100$ & COV = 0.1
				0.93 for $\delta_h/\delta_v = 100$ & COV = 0.2
				Scenario A:
				0.99 for $\delta_h / \delta_v = 1$ & COV = 0.1
				0.88 for $\delta_h/\delta_v = 1$ & COV = 0.3
				0.75 for $\delta_h/\delta_v = 1$ & COV = 0.5
				0.98 for $\delta_h/\delta_v = 10$ & COV = 0.1
				0.88 for $\delta_h/\delta_v = 10$ & COV = 0.3
				0.76 for $\delta_h/\delta_v = 10$ & COV = 0.5
				0.98 for $\delta_h/\delta_v = 30$ & COV = 0.1
				0.88 for $\delta_h/\delta_v = 30$ & COV = 0.3
				0.77 for $\delta_h/\delta_v = 30$ & COV = 0.5
Mobilized shear				0.98 for $\delta_h/\delta_v = 100$ & COV = 0.1
strength of a		Penetration		0.89 for $\delta_h / \delta_v = 100$ & COV = 0.3
deep excavation		depth (H _p)	$0.2 \times H_p$	0.78 for $\delta_h/\delta_v = 100$ & COV = 0.5
in clay				Scenario B:
				0.98 for $\delta_h / \delta_v = 1$ & COV = 0.1
				0.91 for $\delta_h / \delta_v = 1$ & COV = 0.3
				0.82 for $\delta_h/\delta_v = 1$ & COV = 0.5
				0.98 for $\delta_h/\delta_v = 10$ & COV = 0.1
				0.93 for $\delta_h/\delta_v = 10$ & COV = 0.3
				0.85 for $\delta_h\!/\!\delta_v=10$ & COV = 0.5
				0.98 for $\delta_h\!/\delta_v=30$ & COV = 0.1
				0.94 for $\delta_h/\delta_v = 30$ & COV = 0.3
				0.88 for $\delta_h/\delta_v=30$ & COV = 0.5
				0.98 for $\delta_h / \delta_v = 100$ & COV = 0.1

			0.97 for $\delta_h/\delta_v = 100$ & COV = 0.3
			0.93 for $\delta_h / \delta_v = 100$ & COV = 0.5