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DOI
10.1007/978-3-030-55874-1_61

Publication date
2021

Document Version
Final published version

Published in
Numerical Mathematics and Advanced Applications, ENUMATH 2019 - European Conference

Citation (APA)
Kootte, M. E., & Vuik, C. (2021). The Master-Slave Splitting Extended to Power Flow Problems on Integrated Networks with an Unbalanced Distribution Network. In F. J. Vermolen, & C. Vuik (Eds.), *Numerical Mathematics and Advanced Applications, ENUMATH 2019 - European Conference* (pp. 625-632). (Lecture Notes in Computational Science and Engineering; Vol. 139). Springer Science. https://doi.org/10.1007/978-3-030-55874-1_61

The Master-Slave Splitting Extended to Power Flow Problems on Integrated Networks with an Unbalanced Distribution Network

M. E. Kootte and C. Vuik

Abstract An integrated network consists of a transmission network and at least one distribution network which are connected to each other via a substation. One way to do power flow simulations on these integrated networks is the Master-Slave splitting method. This method splits the integrated network and iterates between the separate transmission (the master) and distribution (the slave) network. In this paper, we extend the method to hybrid networks: a network consisting of a balanced transmission and an unbalanced distribution network. An extra handling is necessary to get the Master-slave splitting to work on hybrid networks. We explain two approaches to use the Master-Slave splitting on a hybrid network and compare these approaches on accuracy, computational time, and convergence, by doing test-simulations. The Master-Slave splitting is interesting when distribution and transmission systems have different characteristics, are in geographically distinct locations, or when system operators are not able or allowed to share data of their network with each other. The extension to hybrid networks makes this method generally applicable and an interesting choice to do power flow simulations on integrated networks.

1 Introduction

System operators (SO's) use power flow simulations for safe operation and planning of the electricity grid. In general, a country has one high-voltage transmission network and several medium/low-voltage distribution networks and each SO studies its network separately. The electricity system is changing because of the increasing demand of electricity, the supply of renewable resources, and distributed generation. These changes lead to network interactions that need to be studied with power flow simulations that run on integrated transmission-distribution networks. The different

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F. J. Vermolen, C. Vuik (eds.), *Numerical Mathematics and Advanced Applications ENUMATH 2019*, Lecture Notes in Computational Science and Engineering 139,
https://doi.org/10.1007/978-3-030-55874-1_61

characteristics of the transmission and distribution network require different power flow models and makes integration difficult.

Researchers are paying more attention to integrating power flow models. One of the presented ideas is to unify the power flow models and solve them as a whole [1]. This method has several disadvantages: system operators are not always allowed to share data of their complete network with each other, transmission and distribution systems have different characteristics and require own power flow solvers [2], and the systems are modeled in different units.

Another way to do integrated simulations that overcomes these disadvantages is the Master-Slave splitting (MSS) method [3]. The MSS-method is an iterative method, in which the solution is based on convergence of the voltage mismatch on the boundary between the master (the transmission network) and the slave (the distribution network). At every iteration, the two systems are solved on its own and share only information of the boundary bus with each other. This allows for using power flow algorithms that are appropriate for the specific network conditions and minimizes the data communication between systems. Previous research has shown that the MSS-method has good convergence characteristics when they are applied on a balanced integrated network. However, the distribution network is in general not balanced. Although the authors of the MSS-method describe how the MSS-method is applied on unbalanced distribution networks, they do not test if their method indeed still works.

In this paper, we extend the MSS-method to work on a balanced-unbalanced integrated network and evaluate its behavior on several test-cases. We compare its solution on accuracy and convergence with non-integrated network simulations, i.e. simulations that SO's currently use to study their network separately. In the rest of the paper we describe how we model the power flow problem for balanced and unbalanced networks (Sect. 2), we explain and extend the MSS-method (Sect. 3), we run several numerical simulations and analyze the results (Sect. 4), and draw conclusions from these results (Sect. 5).

2 Characterization of the Power Flow Problem

The steady-state power flow problem is the problem of determining the voltages V in a network, given the specified power $S = P + \iota Q$ and current I [4], ι being the imaginary number. V and I are related by Ohm's Law, $I = YV$, where Y is the admittance. S and V are related by $S = VI^*$. Because the currents are never given in an electricity system, we substitute Ohm's Law into $S = VI^*$ and get a nonlinear equation for S. Power is generated in three phases leading to three sinusoidal functions that describe phase a, b, and c of the voltage, represented by $V = |V|e^{\delta\iota}$ (a magnitude and phase-angle), and of the current. In balanced systems, the phase magnitudes ($|\cdot|$) and angles (ϕ) between two phases are equal: For a voltage V this means that $|V|_a = |V|_b = |V|_c$ and $\phi_{ab} = \phi_{bc} = \phi_{ca} = \frac{2}{3}\pi$. To simplify and speed-up the computations, we only have to model phase a and deduct the other two phases from here. In unbalanced systems, the magnitudes and angles are not equal: All the three-phases are included in the model. The nonlinear power flow equation can be described as follows:

$$S_p = V_p(YV)_p^*, \quad \begin{cases} p \in \{a\}, & \text{balanced systems,} \\ p \in \{a, b, c\} & \text{unbalanced systems.} \end{cases} \tag{1}$$

We represent an electricity network as a graph consisting of buses $i = 1, .., N$ and branches (named after the two buses connecting them). These buses are either a PQ-bus, a PV-bus, or a slack bus, depending on the information we know at that point [5]. We solve equation (1) in an iterative manner for V. All loads in a network are modeled as PQ-buses: Power is consumed at these buses. Generators, buses where power is supplied, are modeled as PV-buses, except for the first generator bus: This is the slack bus. Each network has exactly one slack bus and can have one or multiple PQ and PV-buses. Table 1 describes the known and unknown variables of these buses. We explain in Sect. 3 how we treat the slack buses.

We do power flow simulations in per-unit (pu) quantities and not in engineering quantities. This means that the quantities are scaled by base values such that the voltage is close to unity. This has the advantage that it eliminates erroneous values by scaling them in a narrow range [4].

2.1 *Integrated Networks*

An integrated network consists of a transmission network and at least one distribution network. The separate transmission and distribution networks have distinct characteristics and therefore require own appropriate algorithms. Transmission networks are balanced networks and modeled in single-phase. As the MSS-method solves the two systems on its own, it allows for using a preferred algorithm for each network. We use the Newton-Raphson power mismatch (NR-power) [4] method to solve single-phase transmission networks. Distribution networks are in general unbalanced networks and must be modeled in three-phase. We use the Newton-Raphson three-phase current injection method (NR-TCIM) [6] to solve distribution networks.

Table 1 Bustypes in a network and the information we know and not know at each bus i

Bus type	Known	Unknown
PQ-bus	P_i, Q_i	δ_i, $\lvert V_i\rvert$
PV-bus	P_i, $\lvert V_i\rvert$	Q_i, δ_i
Slack bus	δ_i, $\lvert V_i\rvert$	P_i, Q_i

3 Solving the Power Flow Problem with the Master-Slave Splitting Method

The Master-Slave splitting method [3] is an iterative method that splits the integrated network in a master, the transmission network, and a slave, the distribution network, and solves them on their own. Because the master and slave are solved on its own, they both require a slack bus. One of the load buses of the master is taken as the slack bus of the slave and this bus becomes a direct voltage source for the slave. This load bus is called the boundary B. When multiple slaves are connected to the master, the connecting load buses form the boundary-set $\mathbf{B}$. The voltage source must be equal to the loads in the slave system.

The MSS-method starts by solving the slave: The voltage source from the master is taken as the slack bus for the slave. The slack bus needs the voltage magnitude and angle as known parameter, but as the master is not yet solved, we start with an initial guess of the voltage source, i.e. V_B^0. With this information, we solve the slave with NR-TCIM. We then continue to the master: The boundary bus B is a load-bus for the master, hence we must know active and reactive power $S_B = P_B + \iota Q_B$ at this bus. From the slave, we know S_B and we inject this output into the master. The slack bus, as present in the original transmission network when it is modeled as a separated network, remains the slack bus for the master. This gives us enough information to solve the master, which we do with the NR-power method. Solving the master gives us the voltage V_B. We compare this voltage with the voltage that was previous injected into the slave. When the difference is smaller than a certain tolerance value ϵ, the system has converged. Otherwise, we repeat these steps until we reach convergence. We summarize these steps in Algorithm 1.

Algorithm 1 General algorithmic approach of the Master-Slave splitting method

```
1: Set iteration counter $\nu = 0$. Initialize the voltage $V_B^0$ of the Slave.
2: Solve the slave system. Output: $S_B^{\nu+1}$.
3: Inject $S_B^{\nu+1}$ into the Master.
4: Solve the Master. Output: $V_B^{\nu+1}$.
5: Is $|V_B^{\nu+1} - V_B^{\nu}|_1 > \epsilon$ ? Repeat step 2 till 5.
```

3.1 The Master-Slave Splitting Extended to Balanced/Unbalanced Networks

We are working with a combined balanced-unbalanced system. In order to use the boundary output from the slave and use it as input for the master (and vice-versa), we need to make some modifications. We can do this in two ways: (1) modeling the transmission network in three-phase or (2) transform only the boundary state output such that it matches the input format. If we model the transmission network in three-phase, we integrate two three-phase networks. We call this network an homogeneous network. We keep the assumption that the entire transmission network is balanced. For method (2), we need to transform the three-phase power output S_B^{abc} to a single-phase quantity and the single-phase voltage output V_B^a to a three-phase quantity. We make the assumption that this boundary bus B is balanced. This means that the power injected into the single-phase system is equally influenced by all three-phases:

$$S_B^a = \frac{1}{3}[1\ 1\ 1][S_B^a\ S_B^b\ S_B^c]^T. \tag{2}$$

For the voltage, this means that we can deduct phase b and c, as explained in the beginning of Sect. 2, from phase a:

$$[V_B^a\ V_B^b\ V_B^c] = V_B^a[1\ a^2\ a]^T, \quad a = e^{2/3\pi\iota}. \tag{3}$$

After we received the output in line 2 and 3 of Algorithm 1, we apply transformations (2) and (3) respectively, before we continue to the next line of the algorithm.

The Master-Slave Iterative Schemes

Two iterative schemes to solve the Master-Slave splitting are the Convergence-Alternating-Iterative (CAI)-scheme and the Multistep-Alternating-Iterative (MAI)-scheme [7]. In the CAI-scheme, explicit convergence tolerance is defined for the transmission and distribution system. At each MSS iteration step, the system is solved once its convergence condition is met. Then its output is injected into the other system. In the MAI-scheme, a maximum number of iterations per transmission and distribution system, $I_{T_{max}}$ and $I_{D_{max}}$ respectively, is defined. At each MSS iteration step, the system is solved within this number of subiterations.

4 Numerical Assessment of Integration Methods

We work in the Matpower[1] library where we created 5 test-cases: T9-D13, T118-D37, T3120-D37, T9-2D13, and T9-3D13. The distribution networks are connected via their original slack bus. The connection node of the transmission network is given. Table 2 explains the networks.

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[1] MATPOWER is a package of free, open-source Matlab-language M-files for solving steady-state power system simulation and optimization problems [8].

Table 2 Comparison on number of iterations (for the MSS-method (I_{MSS}) and the necessary iterations per system (I_T and I_D)), and CPU-time of the four different MSS-methods, applied on five test-cases. The star-marked numbers converged to wrong results. In this case, the MAI method required more than 2 subiterations to converge correctly. The lowest CPU times are printed in bold

	MS-homo-CAI				MS-hybrid-CAI				MS-homo-MAI				MS-hybrid-MAI			
	I_{MSS}	I_T	I_D	CPU	I_{MSS}	I_T	I_D	CPU	I_{MSS}	I_T	I_D	CPU	I_{MSS}	I_T	I_D	CPU
	#	#	#	s	#	#	#	s	#	#	#	s	#	#	#	s
T9-D13	4	4	4	**0.247**	4	5	4	0.296	6	2	2	0.254	6	2	2	0.315
T118-D37	4	6	4	0.352	4	5	4	0.368	8*	2*	2*	0.376*	6	2	2	**0.332**
T3120-D13	4	5	3	2.27	4	8	4	0.635	4*	2*	2*	1.56*	4	2	2	**0.458**
T9-2D13	4	4	4	**0.247**	4	4	4	0.306	6	2	2	0.285	6	2	2	0.334
T9-3D13	5	4	4	**0.288**	4	4	4	0.346	7	2	2	0.313	7	2	2	0.415

- The Matpower Transmission 9-bus network connected at node 7 with a IEEE Distribution 13-bus network (T9-D13).
- The Matpower Transmission 118-bus network connected at node 117 with a IEEE Distribution 37-bus network (T118-D37).
- The Matpower Transmission 3120-bus network connected at node 1000 with the 13-bus Distribution network (T3120-D13).

We changed the 13-bus Distribution network to a 10-bus network by deleting the buses that are connected to a regulator. The 37-bus network is originally a balanced distribution network. We changed it to an unbalanced network by shifting 20% of the loads of phase b equally to phase a and c, as explained by Taranto and Marinho [1]. We created two test-cases with multiple Distribution networks: T9-2D13 and T9-3D13, respectively 2 and 3 D13-networks connected to the T9-network.

We run all simulations in Matlab. We set the tolerance value of the MSS-method to $\epsilon_{MSS} = 1e^{-7}$ and the tolerance of the NR-power method and NR-TCIM both as $\epsilon_{NR-P} = \epsilon_{NR-TCIM} = 1e^{-8}$.

Table 2 shows the number of iterations and CPU time to solve integrated networks with four different MSS-methods. At first glance, the results show that all the methods have good convergence characteristics: they converge within a small number of iterations and amount of time. These numbers are comparable for most of the test-cases. If we take a closer look at the bigger test-case, T3120-D13, we see that the MS-homogeneous methods are slower than the MS-hybrid methods. In this bigger test-case, the difference in size of a single-phase and three-phase model becomes more significant and it is thus expected that hybrid methods would perform better. A last remark is that the MS-homo-MAI method did not always converge to the correct results. Therefore, one should be careful here when using this method. Increasing the number of sub iterations of the transmission system leads to better results, but this brings MAI-method closer to the CAI-method.

In Table 3, we compare the outcome of the first test-case, T9-D13, with the output from separated networks T9 and D13. To make generation and load output of the distribution system match, we changed the load at bus 7 in the transmission network, to the total contribution of the loads in the distribution network. To make a fair

Table 3 Comparison on voltage magnitude $|V|$ and angle δ (in radians) of phase a only of the four Master-Slave splitting methods. The first four rows compare the exact values of the boundary bus B in the MSS network with connection bus (bus 7) of the separated transmission network. The last four rows compare the relative differences of the voltage magnitudes and angles with the separated networks. The two networks are compared individually. E.g.: $||\frac{||V|_{MSS}-|V|_T|}{|V|_T}||_\infty$ is the infinity norm of relative difference between the voltage magnitude of the transmission part of the MSS-method and the separated transmission model

	MS-homo-CAI	MS-hybrid-CAI	MS-homo-MAI	MS-hybrid-MAI
$\|V\|^B_{MSS}$	1.0440	1.0440	1.0443	1.0443
$\|V\|^B_{Sep}$	1.0446	1.0446	1.0446	1.0446
δ^B_{MSS}	0.2261	0.2260	0.2271	0.2272
δ^B_{Sep}	0.2308	0.2308	0.2308	0.2308
$\|\|\frac{\|\|V\|_{MSS}-\|V\|_T\|}{\|V\|_T}\|\|_\infty$	5.9E–4	5.6E–4	9.2E–4	9.1E–4
$\|\|\frac{\|\|V\|_{MSS}-\|V\|_D\|}{\|V\|_D}\|\|_\infty$	5.9E–4	5.6E–4	3.4E–4	3.2E–4
$\|\|\frac{\|\delta_{MSS}-\delta_T\|}{\delta_T}\|\|_\infty$	8.2E–2	8.3E–2	6.5E–2	6.3E–2
$\|\|\frac{\|\delta_{MSS}-\delta_D\|}{\delta_D}\|\|_\infty$	2.1E–2	2.2E–2	1.8E–2	1.7E–2

comparison, we multiplied the pu values of the voltage of the separated distribution network by the pu value of the voltage of the connection bus of the transmission network. Because the slack bus of the distribution network is a reference value for the rest of the network and thus it always holds that $V = 1$ pu. If we multiply this value by the value of V_7^T, we receive this as a new reference for the rest of the network.

Table 3 shows that all methods have similar and accurate output compared with the separated systems. It is also clear that the voltage magnitude is more accurate than the voltage angle. To explain this is an interesting follow-up study.

The MSS-method is an excellent choice if one wants to run parallel computations. The amount of communication between two networks is limited, on average 4 iterations, which makes it a suitable option for parallel computing where one master is connected to several slaves, which are all solved in parallel. Real electricity grids are designed like this, with the distribution networks having a size up to millions of buses, which makes parallel high performance computing a necessary choice. In future research, we want to test the four different MSS-methods on realistic size test-cases.

5 Conclusion

We studied the MSS-method applied on hybrid integrated networks. We showed four possible MSS-methods that deal with unbalanced distribution networks. The four different methods were named after the two possible ways we modeled the balanced networks, as a three-phase or as a single-phase network leading to MS-homogeneous and MS-hybrid methods respectively, and after how we put up the iterative schemes, the CAI-scheme and the MAI-scheme. Three out of four methods have shown to be accurate and efficient to run power flow simulations on integrated networks: The MS-homo-CAI, MS-hybrid-CAI, and MS-hybrid-MAI method. They all converged within reasonable amount of time and number of iterations, while obtaining accurate solutions. The MS-hybrid methods showed their speed-up potential when they are applied on bigger test-cases. The MS-homo-MAI method performed not so well over-all: Although the method has shown to be efficient, it does not always converge to the accurate solution. Therefore, we would not recommend to use this one.

With this extension, we showed that the MSS-method can solve integrated balanced/unbalanced networks with different characteristics. The splitting allows for solving the subsystems with their required algorithm and for sharing of information of only one overlapping boundary bus. Furthermore, it has good potential for parallel high-performance computing, which is necessary to do power flow simulations on real integrated networks.

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