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Electromagnetic Marchenko imaging in 1D for dissipative media

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SUMMARY

We present a one-dimensional lossless scheme to compute an image of a dissipative medium from two single-sided reflection responses. One reflection response is measured at or above the top reflector of a dissipative medium and the other reflection response is computed as if measured at or above the top reflector of a medium with negative dissipation which we call the effectual medium. These two reflection responses together can be used to construct the approximate reflection data of the corresponding lossless medium by multiplying and taking the square root in time domain. The corresponding lossless medium has the same reflectors as the dissipative medium. Then the constructed reflection data can be used to compute the focusing wavefield which focuses at the chosen location in subsurface of the dissipative medium. From the focusing function and constructed reflection response the Green's function for a virtual receiver can be obtained. Because the up- and downgoing parts of the Green's function are retrieved separately, these are used to compute the image. We show with an example that the method works well for a sample in a synthesized waveguide that could be used for measurements in a laboratory.

INTRODUCTION

Until now acoustic and elastic Green's function representations have been derived for a virtual receiver inside a lossless scattering medium (Broggini et al., 2012; Wapenaar et al., 2013; Wapenaar et al., 2014; Costa et al., 2014). These representations involve the measured reflection response at one side of the medium and the up- and downgoing parts of the focusing function that focuses at the virtual receiver depth. The focusing function can be obtained from the same representations at the time where the Green's function is zero while the focusing function is not and in 1D without model information (Slob et al., 2014a).

Electromagnetic schemes have also been derived for lossless media for Green's function retrieval and imaging (Slob et al., 2013; Slob et al., 2014b) and for inversion (Slob et al., 2014c). These were assumed inappropriate for the application to surface ground penetrating radar data. For this reason a scheme that can be used for dissipative media has been introduced (Zhang and Slob, 2016). This requires solving two separate Marchenko equations. One for physical dissipative medium and one for the so-called effectual medium. An effectual medium is the same as the physical medium, but with negative dissipation properties. It is therefore a non-physical medium, but is the time-reversed adjoint medium to the physical medium (Wapenaar et al., 2001). The double-sided reflection and transmission responses of the physical medium are used to compute the single-sided reflection response of the effectual medium. With these two single-sided reflection responses, Marchenko equations can be derived for the dissipative and effectual medium. However, solving two Marchenko equations to arrive at a single image is not economical. We present here the improved 1D version of the theory and show how data recorded at surface levels of the physical medium and effectual medium can be combined to construct the reflection data of corresponding lossless medium. This is an appropriate strategy for low-loss media. We then show how the approximate Marchenko equations follow for the corresponding lossless medium using the constructed approximate lossless reflection response. We then briefly discuss aspects of imaging that are different from the known dissipative media scheme and present an example to illustrate the new method.

THEORY

In 1D we use z as spatial variable, the positive axis points downwards and t denotes time. Time and frequency can be interchanged through a Fourier transformation for which we use $E(z,\omega) = \int E(z,t)\exp(-j\omega t)dt$. The medium is assumed homogeneous for $z < z_0$ and $z > z_m$ and heterogeneous for $z_0 < z < z_m$. For this medium scattering data are assumed known from measurements taken at depth levels z_0 and z_m. At any location we can write electric $E(z,\omega)$ and magnetic $H(z,\omega)$ fields in the frequency domain as up- and downgoing wavefields according to Slob et al. (2014c)

$$E(z,\omega) = \left(\frac{\varsigma(z)}{\eta(z)}\right)^{\frac{1}{4}} \left(p^+(z,\omega) + p^-(z,\omega)\right), \qquad (1)$$

$$H(z,\omega) = \left(\frac{\eta(z)}{\varsigma(z)}\right)^{\frac{1}{4}} \left(p^+(z,\omega) - p^-(z,\omega)\right), \qquad (2)$$

In which $\eta = \sigma + j\omega\varepsilon$, σ and ε being the electric conductivity and permittivity, respectively, $\varsigma = j\omega\mu$, with μ being the magnetic permeability, and where p^+ denotes the downgoing and p^- denotes the upgoing wavefield. At any depth level z_i, the reflection response of the medium below that depth level can be written as a fraction combining the electric and magnetic fields

$$R(z_i,\omega) = \frac{\sqrt{\eta(z_i,\omega)}E(z_i,\omega) - \sqrt{\varsigma(z_i,\omega)}H(z_i,\omega)}{\sqrt{\eta(z_i,\omega)}E(z_i,\omega) + \sqrt{\varsigma(z_i,\omega)}H(z_i,\omega)}, \qquad (3)$$

where the depth levels for the medium parameters are taken in the limit of approaching z_i from above. By using the decompositions of equations 1 and 2 we find

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$$R(z_i,\omega)=\frac{p^-(z_i,\omega)}{p^+(z_i,\omega)}, \qquad (4)$$

For later use we introduce a so-called effectual medium (Wapenaar et al., 2001; Zhang and Slob, 2016), which is a non-physical medium but related to the physical dissipative medium. The effectual medium has the same values for permittivity and permeability but its conductivity has opposite sign compared to that of the dissipative medium. The effectual medium can therefore be seen as the time reverse adjoint to the dissipative medium. The medium parameters and wavefields in the effectual medium corresponding to the dissipative medium are denoted with an overbar, e.g. $\bar{E}(z,\omega)$ denotes the electric field in the effectual medium. The medium parameters can be written as $\bar{\eta}(z)=-\eta^*(z)$ and $\bar{\varsigma}=-\varsigma^*$ signifying that the medium is time reversed adjoint of the dissipative medium. The reflection response of the effectual medium below $z=z_i$ can be given in the same form but with all quantities given an overbar.

The total reflection response at z_0 for a source at z_0 in the dissipative medium and effectual medium are given by $R(z_0)$ and $\bar{R}(z_0)$ and defined in a similar way as given in equation 4. Considering that the effectual medium is time reserved adjoint to the dissipative medium the arrival times are the same, while the reflection strengths differ approximately by the same amount compared to the reflection strength of the loss-less medium. The obtained reflection data of the dissipative medium is weaker than that of the lossless medium and those of the effectual medium is approximately stronger by the same amount. We can cancel the attenuation by taking square root of the product of the reflection data $R(z_0)$ of physical medium and $\bar{R}(z_0)$ of effectual medium at each time instant to construct the reflection data $R'(z_0)$ of the lossless medium which has the same reflectors as the dissipative medium.

$$R'(z_0,t)=\mathrm{sign}(R(z_0,t))\sqrt{R(z_0,t)\bar{R}(z_0,t)}, \qquad (5)$$

After multiplying and taking the square root of these reflection data of physical and effectual medium the approximate lossless reflection data has been constructed. For low-loss media this approach results in single-sided reflection data that can be used in standard Marchenko scheme for lossless media. Because the lossless medium has same reflectors as the dissipative one, we can use the lossless Marchenko scheme to obtain an accurate image as function of travel time.

Coupled Marchenko equations

Following Slob et al. (2014c), we directly give the representations of the coupled Marchenko equations

$$f_1^-(z_0,z_i,t)=f_{1;0}^-(z_0,z_i,t) + \int_{-t_d}^{t_d} f_{1;c}^+(z_0,z_i,t')R'(z_0,t-t')dt' \qquad (6)$$

$$f_{1;c}^+(z_0,z_i,-t)=\int_{-t_d}^{t_d} f_1^-(z_0,z_i,-t')R'(z_0,t-t')dt' \qquad (7)$$

$$f_{1;0}^-(z_0,z_i,t)=\int_{-t_d}^{t_d} T_d^{-1}(z_i,z_0,t')R'(z_0',t-t')dt' \qquad (8)$$

Where $R'(z_0,t)$ is the constructed reflection data of the lossless medium, $f_1^+(z_0,z_i,t)$ denotes the downgoing focusing function and $f_1^-(z_0,z_i,t)$ denotes the upgoing one. $T_d^{-1}(z_i,z_0,t)$ denotes the inverse of first arrival of the transmission response, t_d is the traveltime of the first arrival.

Equations 6 and 7 are two coupled Marchenko equations that can be solved for the up- and downgoing focusing wavefields with the aid of equation 8. In 1D no estimate of the first arrival time is needed because it is given by half the recording time. Furthermore, we also do not need to estimate the energy loss along the propagation path from the surface source level z_0 to the virtual receiver level z_i, because we just need to do the lossless Marchenko scheme when we obtain the lossless reflection data by multiplying and taking square root of the dissipative and effectual reflection data.

Imaging

For 1D lossy models the imaging theory has been given by Zhang and Slob (2016). in that case we need to do the lossy Marchenko scheme for the dissipative reflection data and effectual reflection data respectively. However, assuming a lossless medium for the initial estimate of the focusing function causes amplitude errors in the two images. In addition, errors in the initial estimate may result in incomplete focusing and lead to incomplete elimination of multiples. These multiples will then end up in the image. In order to correct for the amplitude error, we need to multiply and take square root of the two images at each time instant, while keeping the sign of one of the two images. For the improved version proposed in this abstract, based on the constructed reflection data, we just need to use the lossless Marchenko scheme to obtain the image. Because the lossless medium corresponding to the physical medium has the same reflectors and propagation velocities as the constructed lossless reflection data, the lossless Marchenko scheme can be used to compute an accurate image. The reflection amplitudes will be incorrect because equation 5 is an approximate relation between the reflection strength of the physical, effectual and lossless media. These three media all have the same values of the electric permittivity and permeability. In the physical medium conductivity is non-negative, in the effectual medium it is non-positive, whereas in the lossless medium, it is zero. For the detailed steps of the lossless Marchenko imaging, Slob et al. (2014c) gave the theory and we use the same methodology here.

NUMERICAL EXAMPLE

To demonstrate the effectiveness of the method in computing the focusing and subsequently retrieving the Green's functions we have performed a one-dimensional modeling test. The model is shown in Table I and has vertical variations in the relative permittivity ε_r and conductivity σ, while the magnetic permeability is assumed to be at the free space value everywhere in the model. The source emits a Ricker wavelet with center frequency of 400MHz. The modeled reflection data of the dissipative medium (red solid) and the corresponding lossless medium (black dashed) are shown in Figure 1(a). We can see that the amplitudes of the reflection data of the dissipative medium being weaker in the deeper part because of the attenuation. Then we modeled the reflection data of the effectual medium (red dashed) and did the comparison with the lossless medium reflection data (black solid), the result is shown in Figure 1(b). We can see that the amplitudes of the reflection data of the effectual medium being stronger in the deeper part because of the negative attenuation. Finally, we multiplied the reflection data of the dissipative and effectual medium and then computed its square root while keeping the sign of the data to cancel the attenuation and obtain the constructed reflection data (red solid) of the approximate lossless medium and the comparison with the modeled lossless reflection data (black dashed) is shown in Figure1(c). We can see that after the processing the attenuation has been canceled and the constructed reflection data of the lossless medium coincides well with the modeled reflection data of the lossless medium. Note that the values for the conductivity in Table I are realistic values. Based on the modeled and constructed reflection data, equations 6 and 7 are solved for the focusing functions using half the recording time as focusing time and then do the imaging using the retrieved Green's functions. Firstly, we use the reflection data of the dissipative and effectual medium to do the lossless Marchenko imaging the results are shown in Figure 2(a), we can see that for the imaging results because of the positive and negative atteuation, the two images are invalid when compared to the exact image (black solid), the internal multiples are not properly eliminated and cause some artifacts in the images. Then we use the reflection data of the dissipative and effectual medium to do the lossy Marchenko imaging. This is followed by multiplying the two images at each time instant and take the square root as value of the finial image. This result (red dashed) is shown in Figure 2(b) together with the exact image of the lossless model. We can see that they are very similar. Finally, we do the improved version proposed in this abstract: 1) multiply the reflection data of the dissipative and effectual medium and take the square root at each time instant to obtain the constructed reflection data of the lossless medium; 2) use the constructed reflection data in the lossless Marchenko imaging. The image (red dashed) is shown in Figure 2(c) and compared with the exact image (black solid). We can see that the image coincides well with the exact image. Comparing Figure 2(b) to Figure 2(c) we can observe that images coincide well with the exact image and no information of multiples in the data is present in these images, although in Figure 2(c) we can see some remnants of multiples in the image. These remnants are very small. However, for the image (red dashed) in Figure 2(b), we need two equations, compute the Green's functions for the dissipative and effectual medium and compute two images after which these two images are combined to form the final image. For the result in Figure 2(c), we combine the reflection responses and need to solve only one Marchenko equation and compute one Green's function and compute a single image. These images in Figure 2(b) and Figure 2(c) have valid amplitudes that allow for further analysis to obtain permittivity and conductivity values in each layer. The scheme proposed in this abstract is almost as accurate as the previous scheme but the results are achieved at half the computational cost.

CONCLUSION

We have shown an improved 1D theory and a numerical example to retrieve the Green's function of a virtual receiver located at a chosen position in the subsurface of a dissipative medium and to compute an image from the Green's function. Compared to the existing lossy Marchenko scheme, the improved scheme changes the dissipative and effectual reflection data to lossless reflection data and uses the lossless Marchenko imaging scheme. The energy loss along the propagation paths has been eliminated by combining the data in such a way that the single-sided lossless medium reflection response is obtained. This has reduced the computational cost by a factor two. In a 3D medium, the effectiveness of this method remains to be investigated.

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TABLE I
Permittivity and conductivity model

$d(m)$	ε_r	$\sigma(mS/m)$
1.2	1	0.1
0.95	6.4	0.9
1	2	0.85
0.9	9	0.8
0.95	12.1	2.5
1	16.1	7.5

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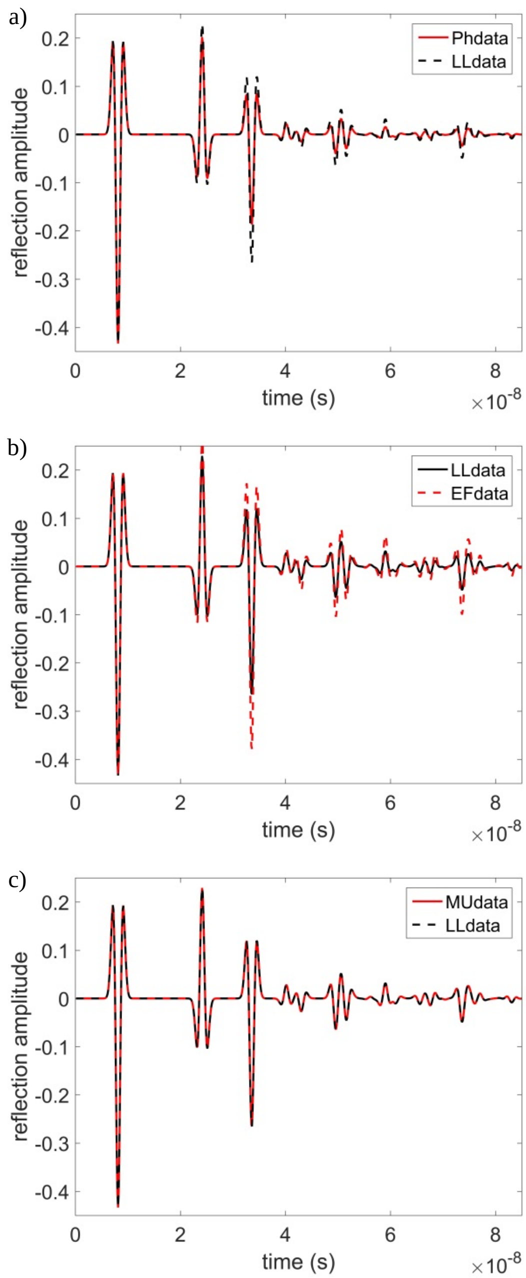

Figure 1: a) The modeled reflection data. The red solid indicates the reflection data of the dissipative medium and the balck dashed indicates the reflection data of the lossless medium. b) the modeled reflection data of the effectual medium (red dashed) and the lossless medium reflection daata (black solid). c) the constructed reflection data (red solid) of the lossless medium obtained by multiplying and taking square root of the reflection data of dissipatvie and effectual mediums at each time instant and the lossless reflection data(black dashed) .

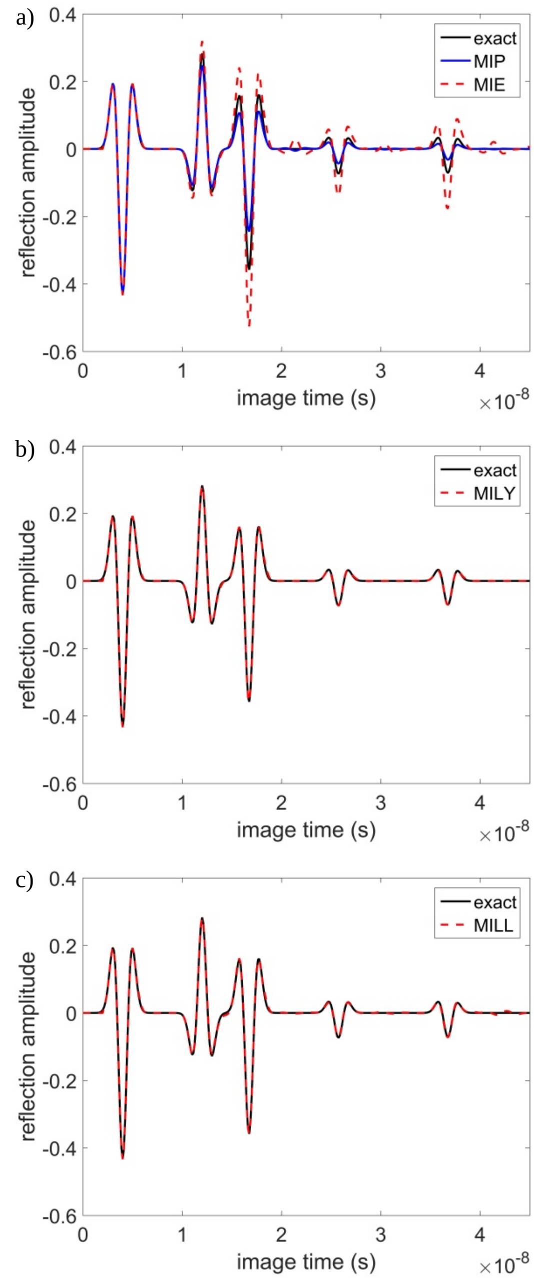

Figure 2: a) the imaging results. The blue solid indicates the image of the dissipative reflection data and the red dashed indicates the image of the effectual reflection data obtained by the lossless Marchenko scheme, the black solid indicates the exact image. b) the image (red dashed) obtained by the lossy Marchenko imaging and the exact image (black solid). c) the image (red dashed) obtained by the lossless Marchenko imaging and the exact image (black solid).